Intangibles, Concentration, and the Labor Share[*]

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September 9, 2021

Abstract

Over the past three decades, the U.S. business sector has been characterized by increasing concentration and decreasing measured labor share. Over the same period, investment in BEA-measured intangible capital, mainly software and R&D, has grown rapidly as a share of total business income. This paper develops a quantitative general equilibrium model of firm dynamics and shows that intangibles play a key role in jointly understanding the trends in measured labor share and concentration, emphasizing the distinct economic properties of intangible capital. The model is consistent with important aspects of firm behavior at the micro level. An intangible-investment-specific technical change (IISTC) shifts the distribution of firms toward large, intangible-intensive firms with low labor shares. When the IISTC is calibrated to match the observed decline in the relative price of intangible investment goods, the model accounts for more than two-thirds of the observed rise in concentration and approximately half of the observed decline in the measured labor share.

Keywords: Intangible Investment, Concentration, Labor Share, Technical Change

JEL Classification: E13, E22, E23, E25, L25, O33

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1 Introduction

Over the past three decades, the U.S. business sector has been characterized by increased industrial concentration in terms of the employment share and market share of very large firms at the national level as well as declined measured labor income share. Over the same period, BEA-measured investment in intangibles, mainly software and R&D, has risen relative to total business income. The relative price of intangible investment goods measured by the BEA has also been declining drastically and secularly. These trends may be linked to each other. Autor, Dorn, Katz, Patterson, and Van-Reenen (2020) find that the rise in industrial concentration and the fall in labor share are positively associated in major sectors of the U.S. However, the potential force that drives this relationship and, more importantly, whether it is quantitatively powerful enough to explain declining labor share and rising concentration together are still open questions.

Intangibles may play a key role in jointly understanding the trends in measured labor share and concentration. First, the decline in the labor share can be driven by some intangible-related technology advances, such as a dramatic fall in the relative price of intangibles, particularly software, due to some potential substitutability between labor and intangible capital. Second, the measurement of labor share depends on the measurement of intangibles: whether capitalizing intangible expenditures or not affects the measurement of national income and thus the labor share (Koh, Santaeulalia-Llopis, and Zheng, 2020; McGrattan and Prescott, 2010b). Moreover, intangible capital, compared to traditional physical capital, has some distinct economic features such as non-rivalry and scalability. These features promote economies of scale, such that firms that are highly productive in producing intangibles become larger, which may have enabled the rise in industrial concentration.

This paper develops a quantitative general equilibrium model of firm dynamics to explore the role of intangibles in explaining the secular change in concentration and measured labor share in the U.S. business sector over the last three decades, emphasizing the link between the micro level heterogeneity and macro level outcomes. Compared to the existing literature that jointly studies the evolution of labor share and concentration, particularly Aghion, Bergeaud, Boppart, Klenow, and Li (2019); Akcigit and Ates (2019); Autor, Dorn, Katz, Patterson, and Van-Reenen (2020); DeLoecker and Eeckhout (2017), the novelty of this paper is twofold. First, it quantitatively exploits the implications of the distinct economic property of intangible capital—that the usage of intangible capital is non-rival in producing different types of goods simultaneously—on concentration and labor share. Due to the non-rivalry property, a proportion of highly productive firms demonstrate increasing

returns to scale technology *a là* Romer (1986). As these firms accumulate more intangible capital, they are able to produce more efficiently, thus growing larger in size, being more intangible capital intensive, and less labor intensive. They benefit more and become even larger, as producing intangibles becomes cheaper. The firm distribution is thus shifted toward large, intangible-intensive firms with low labor shares. This reallocation channel, consistent with the empirical evidence documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2020); Kehrig and Vincent (2021) using micro data, accounts for more than two-thirds of the observed rise in concentration in terms of both the employment share and the market share.

Second, this paper shows that an intangible-investment-specific technical change (IISTC) contributes to a significant part of the decline in measured labor share while simultaneously taking into consideration the measurement concern of labor share discussed in Koh, Santaeulalia-Llopis, and Zheng (2020); McGrattan and Prescott (2010b). More specifically, I show that when the IISTC is targeted to match the decline in the relative price of intangibles from the data, the model accounts for approximately half of the decline in the measured labor share regardless of whether intangible expenditures are treated as final output or not.

My approach is to develop a general equilibrium model of firm dynamics where individual firms produce both consumption/physical investment goods bundles and intangible investment goods, and the former serves as the numeraire in this economy. Firms have heterogeneous persistent productivities in producing numeraire goods and intangible investment goods. Moreover, they differ in technologies, which determine each firm's optimal operating scale. An IISTC is modeled as a permanent increase in the aggregate productivity in producing intangible investment goods relative to that of numeraire goods.

As a production input, intangible capital differs from traditional physical capital in two aspects. First, intangible capital is firm-specific in the sense that each firm accumulates its own intangible capital within the firm. Second, as mentioned at the very beginning, the usage of intangible capital is non-rival in the sense that the intangible capital a firm uses to produce the numeraire good can also simultaneously be used to produce the intangible investment good. Due to the non-rivalry property, firms that are highly productive in producing both types of goods and operate at a sufficiently large scale demonstrate increasing-returns-to-scale technology *a là* Romer (1986). As these firms accumulate more intangible capital, they are able to produce more efficiently, thus growing larger in size and more intangible capital intensive, and more intangible capital intensive firms are also less labor intensive. These firms benefit disproportionately from the IISTC due to a general equilibrium effect:

technical change pushes up the equilibrium wage. Consequently, output and labor are reallocated from small firms with high labor shares to large firms with low labor shares. This leads to both the decline in the aggregate labor share and the rise in concentration.

I also enrich my model with two other features. First, I allow for the endogenous entry and exit of firms. This helps generate distributions of firm size and firm age closer to their empirical counterparts and amplifies the impact of the IISTC on concentration. Second, I introduce financial frictions: incumbent firms cannot issue equity, and they face borrowing constraints that restrict leverage to a multiple of collateralizable assets, as in Evans and Jovanovic (1989). Adding such a financial friction contributes to a realistic firm life cycle. Both features matter for the quantitative results on firm dynamics and concentration driven by the IISTC.

I calibrate the model to the U.S. business sector under the assumption that it was at the steady state in the early 1980s to match a rich set of macro and micro moments. In particular, I discipline the production function to target BEA-measured income shares. I also discipline firm-specific production technology, including firms' productivity processes on producing both numeraire and intangible investment goods as well as their heterogeneity in operating scales, to capture two key empirical facts: (i) more intangible-intensive firms are larger, and (ii) firm size distribution is skewed in the sense that a small proportion of highly productive firms account for a very large share of total employment and total final output.

I then show that the calibrated model can reproduce firm-level cross-sectional predictions that are not targeted directly but are consistent with the micro evidence. The most important prediction is the negative correlation between firm size and firm-level labor share documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2020) using U.S. Census data. Another important prediction is that firms that are highly productive in producing both types of goods and operate at a sufficiently large scale exhibit lower labor shares as they grow over the life cycle. This is consistent with Kehrig and Vincent (2021) who empirically find that the aggregate reallocation of value added toward low-labor-share firms is due to units whose labor share fell as they grew in size.

Finally, I use the calibrated model to quantify the aggregate long-term impact of the intangible-investment-specific technical change, which is disciplined by the observed decline in the relative price of intangible investment goods over the period 1980-2016 from the data. In particular, I compare two steady states: one is the initial steady state calibrated to the early 1980s, and the other is the new steady state after the technical change has occurred.

I find that when the IISTC is calibrated to match the observed decline in the relative price of

intangible investment goods, the model can explain both approximately 60% of the observed decline in the current BEA-measured labor share—when intangibles are capitalized—and the approximately 40% of the observed decline in the measured labor share of the pre-1999 revision—when intangibles are not treated as final output.[1] The decline in the pre-1999-revision-measure of the labor share is purely driven by a reallocation effect: the IISTC shifts firm distribution toward large firms with low labor shares, consistent with the empirical evidence documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2020) and Kehrig and Vincent (2021), while the decline in the current BEA-measured labor share is due to both the reallocation effect and the measurement concern that intangibles, which are capitalized in this measure, are rising relative to the value-added. Moreover, the model accounts for more than two-thirds of the observed increase in the employment share of large firms (with more than 500 employees) and of the increase in the share of total sales going to the top 10% firms. In addition, the model also predicts around one-third of the observed reduction in the annual firm entry rate.

To investigate the mechanisms that lead to the above results, I consider a number of extensions to the baseline model by altering its key features one by one to identify the most essential elements of the model and the deep parameters that drive the key relationships and generate the main results of the paper. The experiments show that the firm-specific and nonrival characteristics of intangible capital—the two key features highlighted in the baseline model—is the most essential element in generating the main results. Parameters contributing to matching the skewed size distribution of firms and the empirical fact that intangible-intensive firms are large matter for the quantitative results. Otherwise, the negative correlation between firm size and firm-level labor share would not be strong enough, and the results on the declined labor share and increased concentration driven by the IISTC would be weakened. Without these two key features of intangible capital, even if the model still features these deep parameters, it would not generate the main results of the paper even qualitatively.

My paper also has one implication for the aggregate elasticity of substitution for factors in production. By numerically deriving the aggregate elasticity of substitution for factors in production, I find that the aggregates of production factors—physical and intangible capital, as well as labor and intangible capital—are more substitutable than those implied by an aggregate Cobb-Douglas pro-

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[1]BEA has done two comprehensive revisions of the national income and product account (NIPA) on the capitalization of intellectual property products (IPP). In 1999, the 11th BEA revision capitalized software expenditures. In 2013, the 14th revision started treat R&D expenditures and artistic originals as investment in the form of durable capital. See Koh, Santaeulalia-Llopis, and Zheng (2020) for a more detailed discussion.

duction function. That is, the aggregation of heterogeneous firms' production mimics the behavior of an aggregate CES production function with elasticity of substitution among factors greater than one.

Related Literature   This paper contributes to the voluminous literature on the evolution of the labor share (see, for instance, Elsby, Hobijn, and Sahin (2013) and Karabarbounis and Neiman (2014) using aggregate data, Autor, Dorn, Katz, Patterson, and Van-Reenen (2020) and Kehrig and Vincent (2021) using micro data, Koh, Santaeulalia-Llopis, and Zheng (2020) for the role of intangibles, DeLoecker and Eeckhout (2017) for rising markups, Autor, Dorn, Katz, Patterson, and Van-Reenen (2020); Barkai (2017) for an increase in concentration that causes increasing profit rates, and Acemoglu and Restrepo (2018) for automation, among others). I contribute to the literature by taking into consideration both the measurement issue of labor share as well as the technological change originating from intangible capital.

My work also relates to a set of recent papers concerning increased concentration in the U.S. Some papers (e.g., Aghion et al. (2019); Barkai (2017); DeLoecker and Eeckhout (2017); Gutierrez and Philippon (2017); Liu, Mian, and Sufi (2019)) argue that rising concentration is due to rising markups and declining competition between firms, while others believe that higher market concentration may not necessarily imply higher market power of firms, consistent with Syverson (2004a,b). Autor, Dorn, Katz, Patterson, and Van-Reenen (2020); Bessen (2016); Crouzet and Eberly (2019); and Hopenhayn, Neira, and Singhania (2019) argue that rising concentration is a result of the expansion of more productive firms, although this expansion is driven by different forces. I propose a new driving force for increasing concentration by utilizing the non-rivalry property of intangible capital, which enables some firms to feature increasing-returns-to-scale technologies as they accumulate more intangible capital in house, that is in line with the latter strand of literature: rising market concentration is the natural consequence of a technical change that favors highly productive, intangible-intensive firms.

For the driving force studied in the paper, Karabarbounis and Neiman (2014), Eden and Gaggl (2018), and Lashkari, Bauer, and Boussard (2019) also consider the decline in the price of investment goods. The difference between my paper and theirs is that they focus on the price of either mainly physical investment or information technology goods that contain tangible capital and thus unable to exploit the implications of the distinct economic properties of intangible capital for labor share and concentration, which means the channels are different between our papers.

My paper is most closely related to Koh, Santaeulalia-Llopis, and Zheng (2020) and Autor, Dorn, Katz, Patterson, and Van-Reenen (2020). I attempt to reconcile these two important empirical papers — one uses macro data and the other uses micro data — in a quantitative general equilibrium framework. Koh, Santaeulalia-Llopis, and Zheng (2020) use national account data (BEA) and show that the measurement of labor share depends on how intangibles are measured and how intangible capital rents are allocated between labor income and capital income. What differentiates my paper from theirs is as follows. Koh, Santaeulalia-Llopis, and Zheng (2020) argue that the labor share would have declined little if investments into intangible capital were treated as expenditures rather than investments for the U.S. economy. However, the accounting treatment of intangibles cannot mechanically explain a decline in the payroll-to-sales ratio, or the rising concentration of sales that Autor, Dorn, Katz, Patterson, and Van-Reenen (2020) find to be correlated with declining labor shares at the industry level using Census data. Moreover, if we focus on the corporate sector, which has a clearer measurement of labor income and profit compared to noncorporate sector, we can see, as in Figure 6, that although the magnitude of the decline in the measured labor share without treating intangibles as final output is smaller, it is still declining. Hence, I develop a framework that can also speak to firm-level patterns and links microlevel heterogeneity to macro outcomes, not only on measured labor income shares but also on concentration and business dynamism.

Autor, Dorn, Katz, Patterson, and Van-Reenen (2020) use micro census data and document a set of key empirical patterns that potentially reveal the channels through which labor share declines and concentration rises, and they further link these patterns to a conceptual, qualitative model. They highlight several potential aggregate shocks that favor highly productive, large firms and would jointly lead to increasing concentration and declining labor share. My contribution is to build a quantitative general equilibrium model that replicates the key empirical findings at the firm level documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2020) and identifies the IISTC as such an aggregate shock that is, more importantly, sufficiently quantitatively powerful to explain the long-term trends in both measured labor share and concentration.

As a cautious remark, the results of my paper do not mean—and are far from implying—that the IISTC is the only driver of the observed trends in measured labor share and concentration. Compared with papers that have a horse race structure (see, for example, Akcigit and Ates (2019)), this paper attempts to jointly explain several empirical facts using one driving force. Hence, I abstract my model from many other factors.[2]

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[2]Potential candidates that may drive one or more trends among labor share, concentration, and intangible in-

Layout The paper is organized as follows. Section 2 describes the model setup and defines a recursive competitive equilibrium. Section 3 discusses calibration and cross-sectional implications respectively. In Section 4, I quantify the aggregate long-run effect of the intangible-investment-specific technical change (IISTC) and inspect the mechanisms that lead to the main results. Section 5 discusses the policy implications. Section 6 concludes the paper.

2 Model

In this section, I build a quantitative framework. My starting point is an equilibrium model of firm dynamics that dates back to the classic competitive settings (Jovanovic (1982); Hopenhayn (1992)). I augment this model in three dimensions. First, I incorporate a production technology with intangible capital as a production input in addition to traditional physical capital and labor *a là* McGrattan and Prescott (2010b) and extend it to a heterogeneous firm setup. Second, the model features two distinct economic properties of intangible capital: (1) intangible capital is firm-specific and (2) the usage of intangible capital is non-rival in the sense that intangible capital can be used to produce different types of goods simultaneously. Third, firms have the option to endogenously enter and exit the market, and once in existence, they face financial constraints.

2.1 Environment

Time is discrete and infinite. There is a continuum of firms that are perfectly competitive in the final goods market and produce both a consumption/physical investment goods bundle y and an intangible investment good x_I. The consumption/physical investment goods bundle serves as the numeraire in this economy. Firms own tangible assets a, accumulate intangible capital k_I within themselves through producing x_I, and rent physical capital k_T subject to a borrowing collateral constraint. Firms have idiosyncratic productivity shocks on producing the two types of goods. The persistent shocks to individual productivity, which, together with endogenous entry and exit, yield heterogeneity in production. Households are identical and infinitely lived. They own firms and supply labor to them.

I abstract the model from industry-level heterogeneity as if there is only one sector in this economy to focus on the impact of firm heterogeneity on the macroeconomy. I consider a stationary

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vestment include lower real interest rate, lower effective corporate tax rate, higher R&D subsidies, higher entry cost, lower knowledge diffusion, rising markups, change in labor force participation, globalization and trade, change in legal forms, among others.

equilibrium without aggregate uncertainty.

2.2 Production Technology

Each individual firm produces a consumption/physical investment goods bundle y (numeraire) and intangible investment goods x_I using two types of capital—physical and intangible capital—and labor according to the following technologies (to differentiate firm-specific variables from variables common to all firms, I index each individual firm by i):

$$y(i) = Az(i) \left[\left(k_{T1}(i)^{(1-\mu)} k_I(i)^{\mu} \right)^{1-\alpha} (l_1(i))^{\alpha} \right]^{\eta(i)} \tag{1}$$

and

$$x_I(i) = A_I z_I(i) \left[\left(k_{T2}(i)^{(1-\mu)} k_I(i)^{\mu} \right)^{1-\alpha} (l_2(i))^{\alpha} \right]^{\eta(i)} \tag{2}$$

Among the parameters, $z(i)$ is a firm-specific productivity shock to the production of numeraire goods, following a Markov process. $z_I(i)$ is a firm-specific productivity shock to the production of intangible investment goods, which is time-invariant and unevenly distributed across firms.[3] $\eta(i)$ is a firm-specific permanent shock to the production scale, with $0 < \eta < 1$. α and μ are parameters on labor share and intangible capital share respectively, which are homogeneous across firms and symmetric in both production functions. The value of A_I represents the aggregate productivity in producing intangible investment goods. I call an increase in A_I relative to the aggregate productivity in producing numeraire goods, A, an intangible-investment-specific technical change (IISTC).

There are two features of the production technology that are key to the main results of the paper. First, intangible capital is firm-specific in the sense that each individual firm accumulates its own intangible capital by producing intangible investment goods in-house in each period. This means that there is no common market where intangible assets can be traded. Without this assumption, firms with the highest productivity in producing intangibles would produce all intangible investment goods in the economy. Consequently, there would always be constant income shares across firms regardless of the firm-level heterogeneity if no other elements, such as overhead labor, are introduced

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[3]A possible source of persistent differences in productivity on producing intangibles is their business processes. Consider Amazon versus Barnes & Noble. While Barnes & Noble relies on its chains of physical stores, Amazon developed an online platform to sell books. The different business processes employed by the two companies determine the different amount of resources they devote to investing on intangibles and their efficiency. Moreover, firms like Amazon have established successful business models and logistics that are evidently hard to copy and reverse engineer. As a robust check, making z_I a persistent shock, following a Markov process as well, does not alter my results. See Table 7 in subsection 4.2.

into the setup. However, since the model works through the reallocation of labor among firms with different labor shares, this is a key assumption. Moreover, this assumption is reasonable in the sense that in the data, a significant fraction of BEA-measured intangible investment is indeed done in-house, such as own-account R&D and own-account software.[4]

Although there is no explicit market price of intangible assets, there is a shadow price of intangible investment goods $x_I(i)$ in terms of the numeraire goods $y(i)$ for each individual firm i, which is determined by:

$$p(i) = \frac{1}{A_I z_I(i)} \bigg/ \frac{1}{A z(i)} \tag{3}$$

Later, I will use these shadow prices when I map my model to the data on the aggregate relative price of intangible investment goods.

Second, intangible capital $k_I(i)$ has a distinct characteristic compared to other factors of production. As an input to both types of goods, intangible capital $k_I(i)$ is not split between producing the two types of goods, $y(i)$ and $x_I(i)$, as is the case for physical capital $k_{T1}(i), k_{T2}(i)$ and labor $l_1(i), l_2(i)$. For example, if one equipment is used to produce a certain consumption good, it cannot be used to development a new software simultaneously. However, an existing software can be used both to produce consumption goods and to develop new softwares at the same time. This non-rivalry property, together with the firm-specificity of intangible capital assumption, breaks the usual constant factor share result with a standard Cobb-Douglas technology. For example, the profit share of each firm in this economy is not simply $(1 - \eta)$ because a firm with high productivity in producing intangible investment goods z_I can accumulate intangible capital more efficiently, thus reaping more intangible capital rents which means larger profit relative to its value-added.

Increasing-Returns-to-Scale Technology Due to the two key features of intangible capital discussed above, the production technology features increasing returns to scale *a là* Romer (1986) with positive intangible capital share μ together with sufficiently large scale parameter η.

Define the total production F of a firm by the sum of the production of y and x_I. That is,

$$F(k_{T1}, l_1, k_I, k_{T2}, l_2) = \underbrace{Az \left[\left(k_{T1}^{(1-\mu)} k_I^{\mu} \right)^{1-\alpha} (l_1)^{\alpha} \right]^{\eta}}_{y} + \underbrace{A_I z_I \left[\left(k_{T2}^{(1-\mu)} k_I^{\mu} \right)^{1-\alpha} (l_2)^{\alpha} \right]^{\eta}}_{x_I}$$

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[4]For example, in the Scientific R&D Services Industry (NAICS 5417), the own-account R&D investment accounts for around 77.6% of the total R&D investment in 2007, and in the whole private business sector, this number is 54.8%. Also, investment in the firm-specific software (custom + own-account) in the private business sector accounts for around 70.4% of the total investment in software averaged across years from 1975 to 2016.

Then we multiply *each* production factor in *both* y and x_I production technologies by γ with $\gamma > 1$, we get

$$F(\gamma k_{T1}, \gamma l_1, \gamma k_I, \gamma k_{T2}, \gamma l_2) = Az \left[\left((\gamma k_{T1})^{(1-\mu)} (\gamma^2 k_I)^\mu \right)^{1-\alpha} (\gamma l_1)^\alpha \right]^\eta + A_I z_I \left[\left((\gamma k_{T2})^{(1-\mu)} (\gamma^2 k_I)^\mu \right)^{1-\alpha} (\gamma l_2)^\alpha \right]^\eta$$

This implies

$$F(\gamma k_{T1}, \gamma l_1, \gamma k_I, \gamma k_{T2}, \gamma l_2) = \gamma^{[(1+\mu)(1-\alpha)+\alpha]\eta} (y + x_I) > \gamma F(k_{T1}, l_1, k_I, k_{T2}, l_2)$$

if $[(1+\mu)(1-\alpha)+\alpha]\eta > 1$.

Note that when the intangible share $\mu = 0$, the increasing returns to scale effect disappears. Given μ, α, the scale parameter η must be large enough to ensure the increasing returns to scale effect. Due to the permanent heterogeneity in η, some firms feature the increasing returns to scale technology, thus able to produce more efficiently as they accumulate more intangible capital, while others do not.

2.3 Financial Frictions

Firms face two financial constraints. First, the capital decision involves borrowing physical capital from financial intermediaries (banks) in intraperiod loans. Due to imperfect contractual enforcement frictions, firms can appropriate a fraction $1/\lambda$ of the capital received by banks, with $\lambda > 1$. To preempt this behavior, a firm renting k_T units of physical capital is required to deposit k_T/λ units of the collateral with the bank. This guarantees that, ex post, the firm does not have an incentive to abscond with the capital. I assume that only tangible assets a can serve as collateral, and likewise, intangible capital k_I cannot be liquidated if the firm exits. The main reason, which is consistent with my model setup, especially the firm-specificity of intangible capital, is that there are limited, and sometimes no markets on which intangible assets can be readily sold to other potential users. Intangible assets are not easily separated from the firms and transferred to other users (e.g. proprietary databases or software, or in-process R&D). The consequence is that business lending against intangible assets is very difficult.[5] Thus, I assume firms face collateral constraints of the form, $k_T \leq \lambda a$.

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[5]Based on the results from Falato et al. (2018) that uses a large sample of syndicated loans to US corporations for which a detailed breakdown of types of collateral used is available, only a very few of them (patents and brands) can be used as collateral and only an extremely small minority of secured syndicated loans (about 3% of total loan value) do use them as collateral.

Second, I assume that firms may only issue equity upon entry: an incumbent must keep nonnegative dividends payments d. The model requires both constraints; otherwise, the collateral borrowing constraint can be easily circumvented.[6] An alternative setup is to introduce costly equity issuance, which would play a similar role.[7]

Introducing financial frictions contributes to a more realistic firm life cycle since it hinders the births of start-ups and slows the expansions of young firms. Otherwise, firms jump to their optimal scale almost immediately after their births. Since a main goal of this paper is to study the impact of a technical change that drives intangible investment on firm dynamics, having a model that features realistic firm distribution and life cycles is key.[8]

2.4 Entry and Exit

I model firm entry and exit based on the standard approaches in the literature.[9] In each period, incumbent firms may exit the economy either because they are subject to an exogenous probability or by their endogenous decisions. Due to financial frictions at the firm-level, individual states of productivity, intangible capital, and tangible assets jointly affect incumbents' endogenous exit decisions. Together with endogenous entry decisions by potential entrants, my model is able to reproduce key moments of firm dynamics.

Entry

At each period, there is a fixed mass of potential entrants M_0. Potential entrants draw a productivity for consumption/physical investment goods z_0 from the ergodic distribution Γ_0. Similarly, they draw

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[6]The non-negative dividend constraint captures two key facts about external equity documented in the corporate finance literature. First, firms face significant costs when issuing new equity, both direct flotation costs (see, for example, Smith (1977)) and indirect costs (see, for example, Asquith and Mullins (1986)). Second, firms issue external equity very infrequently (DeAngelo, DeAngelo, and Stulz (2010)). The specific form of the nonnegativity constraint is widely used in the macro literature because it allows for efficient computation of the model in general equilibrium (See Khan and Thomas (2013), Khan, Senga, and Thomas (2017), Ottonello and Winberry (2018), among others).

[7]Costly equity issuance can be introduced in the form of proportional costs of equity issues (e.g., Begenau and Salomao (2019); Cooley and Quadrini (2001); Gomes (2001); Hennessy and Whited (2005)) and quadratic costs (e.g., Hennessy and Whited (2007)). As a robust check, I allow part of the firms (corresponds to public firms) to issue equity at a cost and find there is almost no change in the quantitative results. See Table 7 in subsection 4.2.

[8]From this perspective, the assumption that only tangible assets can be used as collateral is not essential. It is made to be consistent with the empirical evidence. Making intangible capital as collateralizable as tangible assets will not alter the results on concentration and firm dynamics (See Table 7 in subsection 4.2 for a robust check). There are other set-ups that may also be able to achieve this goal. For example, we can make physical capital accumulated rather than rented and introduce physical capital adjustment cost instead of the collateral constraint.

[9]Hopenhayn (1992) is the seminal work in this literature with industry dynamics driven by firms' endogenous entry and exit.Clementi and Palazzo (2016) modify the timing of entry in the Hopenhayn model to investigate the business cycle implications of firm dynamics. I follow an approach similar to them, while introducing the exogenous exit as in Khan and Thomas (2013).

a productivity for intangible investment goods z_I as well as a scale parameter η from distributions Γ_{z_I} and Γ_η respectively. They start with zero intangible capital, i.e., $k_{I0} = 0$, and the same positive amount of tangible assets a_0, which is financed by an equity injection from households. This is the only time when firms are able to issue equity in the benchmark model.

Firms, after observing their draws, decide whether to enter the market by paying the fixed entry cost κ_e, denominated in labor units, which can be interpreted as labor utilized for entry, such as entrepreneurs in start-ups. Since firms start with zero intangible capital, they need to invest in intangible capital in the first period they enter the market given their initial states, (z_0, z_I, η), and start producing both types of goods in the following period. Let $e(z_0, z_I, \eta) \in \{0, 1\}$ denote the entry decision rule.

The value of entry is:

$$v^e(z_0, z_I, \eta) = \max_{k_I'} - k_I' + \beta \int_{\mathcal{Z}} \frac{U'\left(C'\right)}{U'(C)} v^0\left(k_I', (1+r)a_0, z', z_I, \eta\right) \Gamma\left(dz', z\right) \qquad (4)$$

where $v^0(\cdot)$ is the value of an operating firm at the beginning of each period, a function of individual states (k_I, a, z, z_I, η), and $\Gamma\left(dz', z\right)$ is the conditional distribution of z. Since any individual firm is owned by a representative household in the economy, the household's stochastic discounting factor $\frac{U'\left(C'\right)}{U'(C)}$ will appear in the firm's optimization problem. Note that the initial investment in intangibles for start-ups is different from the intangible investment made by incumbents. Incumbents accumulate intangible capital by producing intangible investment goods based on the existing intangible capital stock, while entrants, starting with zero intangible capital, make a one-time investment in intangibles against their future value, based on their initial states (z_0, z_I, η).

The firm chooses to enter the market if and only if the value of entry exceeds start-up costs κ_e, denominated in labor units, plus household equity injection a_0. In other words, potential entrants solve the following entry problem:

$$\max_{e \in \{0,1\}} \left\{ (1-e) \cdot a_0, e \cdot [v^e(z_0, z_I, \eta) - w\kappa_e] \right\} \qquad (5)$$

Entrants start with relatively low net worth compared to mature firms. Due to the fixed start-up cost denominated in labor units, an increase in the wage rate w (resulting from technological advances) may suppress firm entry.[10]

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[10] This is in line with Jo and Senga (2019). They show that in a general equilibrium set-up similar to my framework where heterogeneous firms face credit constraints, increased factor prices, due to credit subsidies, reduce the number

Exit

At the beginning of each period, firms are informed of their respective exit status prior to production. First, there is a fixed probability of exit, π_d, that is common across firms. The remaining firms that survive from this exogenous death shock need to pay κ_o units of labor to continue operating in the next period. This fixed cost of operations denominated in labor can be interpreted as overhead labor, such as expenses on hiring HRs and accountants, which creates a binary exit decision. If a firm does not pay this cost, it has to exit the economy permanently with a liquidation value equal to its net worth in terms of tangible assets a. Thus, only firms continuing to the next period make intertemporal decisions on investment and savings after paying κ_o labor units. Due to the fixed operation cost, denominated in labor units, an increase in the wage rate will have a larger impact on small firms. This is relevant for quantitative results, as I show in Section 4.

Both endogenous and exogenous exit are necessary elements of this model. With a fixed probability of exogenous exit, all firms have an equal chance to exit, regardless of firm size. This assumption helps the model reproduce the empirical distribution of firm size and firm age by allowing turnover among large, mature firms (e.g., job destruction in large and mature firms). The endogenous exit margin of firm dynamics enables relatively less profitable firms to endogenously choose to exit. Financially constrained and unproductive firms are more likely to exit and have higher job destruction rates. This is consistent with the empirical evidence that small and young firms have higher exit and job destruction rates in general.

Let $v^0(\cdot)$ be the value of an operating firm at the beginning of the current period before it is known whether it survives the exogenous exit. Accordingly, define $v^1(\cdot)$ as a surviving firm's value, before it decides to pay the operation cost κ_o in terms of labor. If a firm decides to continue to the next period, its value is given by $v(\cdot)$. Once a firm exits, its liquidation value equals its net worth in terms of tangible assets a. Let $ex(k_I, a, z, z_I, \eta) \in \{0, 1\}$ denote the endogenous exit decision rule. Then, firms' value can be written as:

$$v^0(k_I, a, z, z_I, \eta) = \pi_d \cdot a + (1 - \pi_d) v^1(k_I, a, z, z_I, \eta) \tag{6}$$

$$v^1(k_I, a, z, z_I, \eta) = \max_{ex \in \{0,1\}} \{(ex \cdot a, (1 - ex)v(k_I, a, z, z_I, \eta)\} \tag{7}$$

Before describing incumbent firms' problem in detail, I outline the precise timing of the model,

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of firms in production.

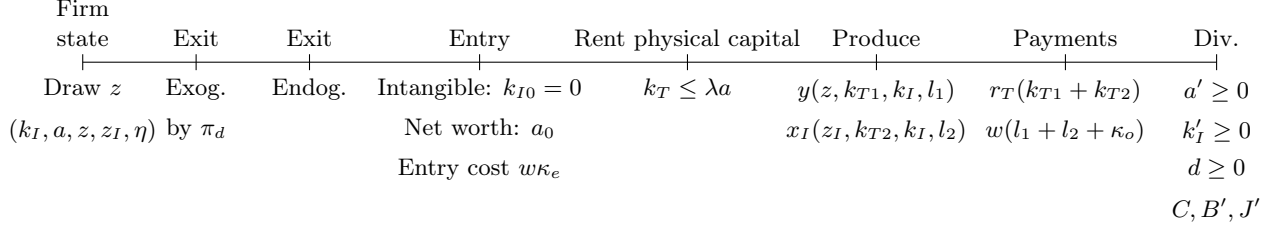| Firm state | Exit | Exit | Entry | Rent physical capital | Produce | Payments | Div. |

Figure 1: Timeline of the Model

summarized in Figure 1. Within a period, the events unfold as follows: (i) realization of the productivity shocks for incumbent firms; (ii) endogenous and exogenous exit of incumbents; (iii) realization of initial productivity and entry decisions of potential entrants; (iv) production and revenues from sales; (v) payment of wage bills, operation expenses, and physical capital rental costs; and (vii) firm decisions on dividend payment, intangible capital, and tangible assets for the next period, and household consumption/saving decisions.

Incumbent Firm

The recursive form of the problem of incumbent firms is given by

$$v\left(k_I, a, z, z_I, \eta\right) = \max_{k_I', a', l_1, l_2, k_{T1}, k_{T2}, d} \left\{ d + \beta \int_{\mathcal{Z}} \frac{U'\left(C'\right)}{U'\left(C\right)} v^0\left(k_I', a', z', z_I, \eta\right) \Gamma\left(dz', z\right) \right\} \quad (8)$$

s.t.

$$\underbrace{d}_{dividend} + \underbrace{px_I}_{intangible\ invest.} + a' = \underbrace{y + px_I}_{NIPA\ income} - \underbrace{wl}_{wage} - \underbrace{w\kappa_o}_{overhead\ labor} - \underbrace{(r + \delta_T)k_T}_{rental\ cost} + (1 + r)\,a$$

$$y = Az\left[\left(k_{T1}^{(1-\mu)}k_I^{\mu}\right)^{1-\alpha}(l_1)^{\alpha}\right]^{\eta}$$

$$x_I = A_I z_I \left[\left(k_{T2}^{(1-\mu)}k_I^{\mu}\right)^{1-\alpha}(l_2)^{\alpha}\right]^{\eta}$$

$$k_I' = (1 - \delta_I)\,k_I + x_I, x_I \geq 0$$

$$k_T = k_{T1} + k_{T2}, l = l_1 + l_2$$

$$k_T \leq \lambda a, d \geq 0$$

Note that there is irreversibility in intangible investment since the production of x_I is always nonnegative. This is consistent with two facts: intangible assets are firm-specific and there is no market where they can be traded. Additionally, the value-added of a firm is defined as $y + px_I$ to be consistent with the post-2013 BEA-NIPA measure. Moreover, due to the financial constraints, the effective physical capital rental rate r_T is at least as large as $(r + \delta_T)$.

To help understand the budget constraint and preface how I take the model to the data, firm debt is defined by the identity $b := k_{T1} + k_{T2} - a$, with the understanding that $b < 0$ denotes savings. Making this substitution reveals an alternative formulation of the model in which the firm owns its physical capital rather than renting it and faces a constraint on leverage: $b \leq \theta k_T$ where $\lambda = 1/(1 - \theta)$. With the state vector $(k_I, k_{T1}, k_{T2}, b, z, z_I)$, the firm faces the following budget constraint:

$$\underbrace{d}_{dividend} + \underbrace{x_{T1} + x_{T2}}_{physical\ investment} + \underbrace{px_I}_{intangible\ investment} = \underbrace{y + px_I}_{NIPA\ income} - \underbrace{wl}_{wage} - \underbrace{w\kappa_o}_{overhead\ labor} - r_t b_t + \underbrace{b_{t+1} - b_t}_{\triangle Borrowing}$$

2.5 Representative Households

Assume that there is a unit measure of identical households in the economy. In each period, households consume, supply labor inelastically, and invest in one-period risk-free bonds and firms' shares:

$$W(B, J) = max_{B', J', C>0} U(C) + \beta W\left(B', J'\right) \tag{9}$$

subject to

$$C + B' + QJ' = w\bar{N} + (D + Q)J + (1 + r)B$$

where C is consumption by households, B are one period risk-free bonds, J are shares of the mutual fund composed of all firms in the economy, and D are aggregate dividends per share. The household takes as given the return of risk-free bonds $(1 + r)$, the share price Q, and the price of the consumption, which is the numeraire, so normalized to 1. In steady states, from the first-order conditions for deposits and share holdings, I obtain $1/(1 + r) = \beta$ and $Q = \beta(Q + D)$, which implies a time-invariant rate of return of $r = \beta^{-1} - 1$ on both bonds and shares. The household is therefore indifferent over portfolios. For simplicity, I assume $U(C) = C$. Because of risk neutrality, households are indifferent over the timing of consumption as well.

2.6 Stationary Equilibrium

The state space for an incumbent firm is $S \equiv (\mathcal{K}_I \times \mathcal{A} \times \mathcal{Z} \times \mathcal{Z}_{\mathcal{I}} \times \mathcal{H})$ where $(k_I, a, z, z_I, \eta) \in S$. To simplify the exposition of the equilibrium, it is convenient to use $s \equiv (k_I, a, z, z_I, \eta)$ and $s_0 \equiv (z_0, z_I, \eta)$ as the argument for incumbents' and entrants' decision rules. Also, denote with φ the stationary measure of incumbent firms at the beginning of the period, following the draw of firm-level persistent productivity, before the exogenous exit shock. Accordingly, denote φ^e as the mass of actual entrants. Denote φ^p as the distribution of producing firms that survive from exogenous shocks and decide to continue, and denote φ^{ex} as the distribution of exiting firms (including both exogenous and endogenous).

Definition A **stationary recursive competitive equilibrium** is a collection of firms' decision rules $\left\{k'_I(s), a'(s), d(s), k_{T1}(s), k_{T2}(s), l_1(s), l_2(s), ex(s), e(s_0)\right\}$, value functions $\left\{v^0, v^1, v, v^e\right\}$, a measure of entrants φ^e, a distribution of firms φ, wage w, policy functions for households $\left\{C, B', J'\right\}$ with the associated value function W that solve the optimization problems and clear markets in the following conditions.

1. The decision rules $\left\{k'_I(s), a'(s), d(s), k_{T1}(s), k_{T2}(s), l_1(s), l_2(s), ex(s), e(s_0)\right\}$ solve the firm's problems (5), (6), (7), and (8), $\left\{v^0, v^1, v, v^e\right\}$ are the associated value functions, and φ^e is the mass of entrants implied by

$$\varphi^e = M_0 \int_{\mathcal{Z}} \int_{\mathcal{Z}_{\mathcal{I}}} \int_{\mathcal{H}} e(s_0)\, d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0 \tag{10}$$

2. W solves (9), and $\left\{C, B', S'\right\}$ are the associated policy functions.

3. Labor markets clear:

$$\bar{N} = \int_S (l_1(s) + l_2(s) + \kappa_o)\, d\varphi^p + \int_S \kappa_e d\varphi^e \tag{11}$$

4. Goods markets clear (resource constraints hold):

$$C + K_T - (1 - \delta_T) K_T + M_0 \int_{\mathcal{Z}} \int_{\mathcal{Z}_{\mathcal{I}}} \int_{\mathcal{H}} a_0 \cdot e(s_0)\, d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0 - \int_S a(s) d\varphi^{ex} = Y \tag{12}$$

where $Y = \int_S y(s) d\varphi^p$ and $K_T = \int_S k_T(s) d\varphi^p$

16

5. Shares markets clear (by Walras' Law) at $S = 1$ with share price

$$Q = \int_S v\left(s\right) d\varphi + M_0 \int_{\mathcal{Z}} \int_{\mathcal{Z}_{\mathcal{I}}} \int_{\mathcal{H}} e\left(s_0\right) v^e\left(s_0\right) d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0$$

and aggregate dividends

$$D = \pi_d \int_S a(s) d\varphi + (1 - \pi_d) \int_S \left\{\left[1 - ex\left(s\right)\right] d\left(s\right) + ex\left(s\right) a(s)\right\} d\varphi - M_0 \int_{\mathcal{Z}} \int_{\mathcal{Z}_{\mathcal{I}}} \int_{\mathcal{H}} a_0 \cdot e\left(s_0\right) d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0$$

6. The distribution of firms, φ, is a fixed point where its transition is consistent with the policy functions and the law of motion for φ, which is given by

$$\begin{aligned}
\varphi\left(\mathcal{K}_I \times \mathcal{A} \times \mathcal{Z} \times \mathcal{Z}_{\mathcal{I}} \times \mathcal{H}\right) &= (1 - \pi_d) \int_S \left[1 - ex\left(s\right)\right] \mathbf{1}_{\left\{k'_I(s) \in \mathcal{K}_I\right\}} \mathbf{1}_{\left\{a'(s) \in \mathcal{A}\right\}} \Gamma\left(\mathcal{Z}, z\right) d\varphi \\
&\quad + M_0 \int_{\mathcal{Z}} \int_{\mathcal{Z}_{\mathcal{I}}} \int_{\mathcal{H}} e\left(s_0\right) \mathbf{1}_{\left\{k'_I(s_0) \in \mathcal{K}_I\right\}} \mathbf{1}_{\left\{a'(s_0) \in \mathcal{A}\right\}} \Gamma\left(\mathcal{Z}, z\right) d\Gamma_\eta d\Gamma_{z_I} d\Gamma_0
\end{aligned} \tag{13}$$

3 Calibration and Cross-sectional Implications

I numerically solve the model by using nonlinear methods, and find a stationary equilibrium where individual decisions are consistent with market clearing prices. In subsection 3.1, I discuss how I map from the model to the data. In subsection 3.2, I calibrate the model to be consistent with observed data for the U.S. business sector. Once the model is calibrated, in subsection 3.3, I explore the main cross-sectional implications of the calibrated model, which will inform the aggregate results in Section 4.

3.1 Variables of Interest

Key moments for parameterizing the model are the implied aggregate intangible investment (in terms of numeraire goods), income shares and the distribution of firms.

Price   First, the intangibles considered in this paper are those measured by the BEA, including software, R&D, and artistic originals. There are potentially other types of intangibles such as organizational capital and human capital as emphasized in Corrado, Hulten, and Sichel (2005), but due to the measurement issue, they are not considered in this paper.

Aggregate intangible investment in terms of numeraire goods in the model is defined by aggregating all the individual firms' production of intangible investment goods x_I evaluated at the firm-specific shadow price p. That is, $PX_I = \int_S p(s) x_I(s) d\varphi^p$, where P is the aggregate relative

price of intangible investment goods. Here, I do not derive P explicitly but take it together with the aggregate quantity of intangible investment X_I. Note that intangibles in the model have no price, since intangible capital is accumulated in-house, and thus there is no market where it can be traded. In the data, a large portion of intangible investment is indeed produced in-house and not sold in the market. Because own-account software and R&D are not sold in the market, the BEA estimates the own-account production of software and R&D as the sum of costs of production (i.e., wages, nonwages, and intermediates) plus a markup (Crawford et al., 2014; Moylan, 2001).[11] The best way to infer a "price" from the model and map it to BEA data is working through each individual firm's shadow price p, which is determined by the inverse of the relative productivity in producing intangible investment goods of that firm.

Measured Labor Share I compute two different measures of the labor income share, depending on whether BEA treats intangibles as expenditures or investments. To be consistent with BEA's definition post-2013-revision, own-account intangibles (including software, R&D, and artistic originals) are considered as final output That is, the final output for BEA's definition post-2013-revision should be represented by $Y + PX_I = \int_S y(s)d\varphi^p + \int_S p(s)x_I(s)d\varphi^p$ in the model. Hence, the labor income share (post-2013-revision) in the model is defined as labor compensation divided by the gross value-added of the domestic business sector, taking into intangibles as final output:

$$S_N = \frac{w\bar{N}}{Y + PX_I} = \frac{w\left[\int_S \left(l_1(s) + l_2(s) + \kappa_o\right)d\varphi^p + \int_S \kappa_e d\varphi^e\right]}{\int y(s)d\varphi^p + \int p(s)x_I(s)d\varphi^p} \tag{14}$$

where P is the aggregate price of intangible investment goods in terms of consumption/physical investment.

Correspondingly, the labor income share (pre-1999-revision) when own-account intangibles are not treated as final output in the model is defined as:

$$S_{N_{pre}} = \frac{w\bar{N}}{Y} \tag{15}$$

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[11]There are three types of software: (i)-(ii) prepackaged and custom software, which are estimated from the I-O accounts that are based on receipts for software products; and (iii) own-account software, which is calculated by multiplying the number of programmers times an estimate for the share of time they spend doing tasks associated with non-embedded software development, times a national median wage rate for programmers, times various factors that cover nonwage compensation costs and intermediate inputs, based on BLS employment-by-industry data. For R&D, there are two types: (i) purchased R&D, which is funded by one entity, but produced by another entity; and (ii) own-account R&D, which is produced for an entity's own use. Similar to how BEA estimates software, market prices are used to value purchased R&D, while the value of own-account R&D is estimated as the sum of production costs.

For aggregate measured income shares, I focus on the corporate sector for two reasons: first, it has clearer measurement of labor income and profit compared to non-corporate sector.[12] Second, the way I model firms is more consistent with (non-financial) corporations.

Distribution of Firms For the distribution of firms, I focus on all the employer firms in the U.S. business sector of which the corporate firms are just a subset.[13] The way I deal with this discrepancy is as follows. Since corporations are usually very large firms,[14] in the model, I filter corporate firms based on firm size in terms of final output and choose a cutoff for $y + px_I$, call it $\bar{v}$, such that the total income generated by all the firms with $y + px_I \geq \bar{v}$ divided by the total income generated by all the firms in the economy (i.e. $Y + PX_I$ in equation (14)) equals the corporate income as a share of the domestic business income from BEA-NIPA.[15] Any moments generated from my model that are used to calibrate parameters to match their empirical counterpart from Compustat also use this filtering criterion. In addition, I compare the national account generated from the calibrated model with the BEA-NIPA. The two are close to each other (see Table 9 in Appendix A).

To ensure better estimates and consistency for intangible investment, the starting point of the time-series considered in the paper is 1975 because it is the first year that the Federal Accounting Standards Board (FASB) required firms to report R&D.

3.2 Parameterization

I begin with the subset of parameters calibrated externally, and then consider those estimated within the model. Data moments are averages over 1980-1985 unless otherwise specified.[16] For empirical

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[12]For the corporate sector, labor income is unambiguously defined as the compensation of employees, but for the non-corporate sector such as partnership or sole proprietorship, since owners also use their own time to contribute to the production, part of the proprietor's income should be allocated to labor income. The exact magnitude of the decline in labor income is affected by the treatment of the labor portion of proprietor's income (See Gollin (2002), Elsby, Hobijn, and Sahin (2013)). Moreover, most of the intangible investment in the corporate sector is financed by capital owners, thus contributing to capital income rather than labor income, which is in contrast to non-corporate sector where lots of intangibles or "sweat equity" are financed by workers (Bhandari and McGrattan, 2019; McGrattan and Prescott, 2010b).

[13]The reason why I focus on the distribution of all employer firms is that the data on firm distribution is only available for all the employer firms rather than for firms in corporate sector or for all the firms in the private sector.

[14]For example, based on Dyrda and Pugsley (2019) that uses U.S. census data, 20% of all firms are corporations, accounting for 90% of all sales in the early 1980s.

[15]Admittedly, there also exists a discrepancy between the coverage of BEA-business sector, which covers both employer and non-employer firms, and the coverage of BDS/LBD, which only covers employer firms. However, since non-employer firms do not contribute to the total employment and only take a very small portion in terms of sales (which is 2.48% based on SBO 2007), I assume they cover the same firms. See Table 12 for a comparison among the datasets for the coverage of firms.

[16]Ideally, I would only use data on firms, since financial constraints apply at the firm level. However, since some moments are only available at the establishment-level and my model indeed does not differentiate between firms and establishments, I use firm data whenever I have a choice, and establishment data only when firm-level data is not

Parameter	Description	Value	Source/Target
β	Discount factor	0.960	Annual risk-free rate = 4%
M_0	Mass of potential entrants	0.076	Measure of incumbents = 1
$\bar{N}$	Size of labor force	2.884	Average firm size (BDS)
δ_T	Physical capital depreciation rate	0.050	BEA fixed asset tables
δ_I	Intangible capital depreciation rate	0.215	BEA fixed asset tables
ρ_z	Persistence of prod. process z	0.750	Foster, Haltiwanger, and Syverson (2008)

Table 1: Parameter values set externally

moments related to concentration (e.g., employment share and market share of large firms), what I report is the weighted average by industry since the model in this paper abstracts from industry-level heterogeneity.

3.2.1 Externally Calibrated Parameters

The model period is one year. I set $\beta = 0.960$ to target a risk-free rate of 4%. Since the measure of potential entrants M_0 scales the distribution of entrants φ_e (see equation 13), I choose M_0 to normalize the total measure of incumbent firms to 1. Given a measure 1 of firms, I fix the labor force $\bar{N}$ to target the average firm size in the data. I use BEA fixed asset tables that include both flows and stocks to compute the depreciation rate of physical capital $\delta_T = 0.05$ as well as the depreciation rate of intangible capital $\delta_I = 0.215$ (see Appendix A for more details). The persistent firm-specific productivity in producing a consumption/physical investment goods bundle, z, follows a Markov process. I assume that z is drawn from a time-invariant distribution, $G(z; z^L, z^H, \gamma^z)$, which is a bounded Pareto distribution. In each period, a firm retains its previous level of productivity in producing numeraire goods with a fixed probability ρ_z. I directly use the estimate from Foster, Haltiwanger, and Syverson (2008), which uses Census data on all the employer firms in the U.S. for the persistence of the firm-level productivity process.[17] The remaining parameters that govern the process of z are calibrated within the model. Table 1 summarizes these parameter values.

3.2.2 Internally Calibrated Parameters

Table 2 lists the remaining 18 parameters of the model; these are set by minimizing the distance between a set of empirical moments that are used to discipline these parameters and their equilibrium

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available.

[17]The reason why I can directly use Foster, Haltiwanger, and Syverson (2008)'s estimates is that the firm distribution that I focus on in this paper is also all the employer firms in the U.S.

Parameter		Value	Moment	Data	Model
Production Technology - consumption/physical investment goods & intangible investment goods					
Labor share	α	0.690	Labor compensation/value-added	0.640	0.640
Intangible capital share	μ	0.250	Intangible investment/value-added	0.032	0.032
Permanent Productivity on intangible invest. goods $z_I \in \left\{z_I^L, z_I^H\right\}$					
High - low gap	z_I^H/z_I^L	4.000	Share of sales going to top 10%	0.519	0.463
Mass: z_I^H firms	$\mu_{z_I^H}$	0.050	Intangible-intensive firms		
Persistent Productivity on Consumption/Physical Invest. Goods $z \sim$ **bounded Pareto** $G\left(z; z^L, z^H, \gamma_z\right)$					
Lower bound z	z^L	0.250			
Upper bound z	z^H	7.690			
Shape parameter	γ^z	1.750			
Scale Parameter					
Low DRS in prod.	η_L	0.775	firm size distribution (BDS)	see Table 3	
Mid DRS in prod.	η_M	0.825			
High DRS in prod.	η_H	0.930			
Mass: η_L firms	μ_L	0.700			
Mass: η_H firms	μ_H	0.100			
Entrants					
Initial tangible asset	a_0	1.210	Start-up debt/value-added rel. to aggregate debt/value-added	1.739	1.560
Initial productivity (mean)	$\hat{z}_0$	0.393	Average start-up size rel. to average incumbent size	0.296	0.335
Financial Friction					
Collateral parameter	λ	6.500	Aggregate debt/value-added	0.880	0.912
Entry and Exit					
Exog. exit prob.	π_d	0.003	5-year survival rate	0.485	0.533
Entry cost	κ_e	1.333	Annual entry rate	0.125	0.136
Operating cost	κ_o	0.105	Exit rate $(size < 20)$/exit rate $(size > 500)$	32.11	37.67

Table 2: Parameter calibrated internally

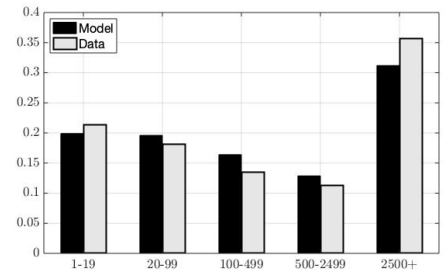|	Population Share (%)		Employment Share (%)	
Employees	Data	Model	Data	Model
1 to 19	88.97	88.97	21.35	19.90
20 to 99	9.30	9.30	18.19	19.63
100 to 499	1.41	1.41	13.45	16.42
500 to 2499	0.24	0.24	11.27	12.85
2500+	0.08	0.08	35.74	31.20

Table 3: Firm Size Distribution: Model v.s. Data (p.p.)

counterparts in the model.[18] The table also lists the targeted moments, their empirical values, and their simulated values from the model. Even though every targeted moment is determined simultaneously by all parameters, in what follows, I discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power.

There are two key sets of parameters to be calibrated within the model: those that characterize the production technology and those that characterize the heterogeneity across firms in the corresponding state variables of productivity. I start by disciplining the parameters for the production technology for both numeraire goods y and intangible investment goods x_I. Recall that

$$y = Az \left[\left(k_{T1}^{(1-\mu)} k_I^\mu \right)^{1-\alpha} (l_1)^\alpha \right]^\eta$$

and

$$x_I = A_I z_I \left[\left(k_{T2}^{(1-\mu)} k_I^\mu \right)^{1-\alpha} (l_2)^\alpha \right]^\eta$$

Because I do not have additional information on whether, and the degree to which, these inputs are substitutes or complements at the firm level, following the literature involving intangible capital as a production input,[19] I assume both technologies take a Cobb-Douglas structure and are symmetric in the sense that they have exactly the same parameters for the factor shares, i.e., α, μ, and η. The labor share α and the intangible capital share μ are chosen so that the model predictions for labor compensation as a share of gross value-added and for intangible investment as a share of gross value-added are consistent with the BEA data.[20]

The other key set of parameters is calibrated to capture two empirical facts: (i) intangible-intensive firms are large ones; and (ii) firm size distribution is highly skewed, for two main reasons. First, a major goal of this paper is to study the impact of a technological change on the distribution of firms, so having firm distribution matched to the data at the initial steady state is important. More importantly, how well this model is able to match the firm distribution determines how well it

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[18]Specifically, the vector of parameters Ψ is chosen to minimize the minimum-distance-estimator criterion function $f(\Psi) = (m_{data} - m_{model}(\Psi))' W (m_{data} - m_{model}(\Psi))$, where m_{data}, m_{model} are the vectors of moments in the data and model, and $W = diag(1/m_{data}^2)$ is a diagonal weighting matrix.

[19]For example, in McGrattan and Prescott (2010a), McGrattan and Prescott (2010b), and Bhandari and McGrattan (2019), they all have intangible capital as a production input and they assume that firms produce at Cobb-Douglas technology because there is no sufficient information from the data to estimate accurately the elasticity of substitution among the factors, i.e. physical capital, intangible capital, and labor, in production.

[20]In my model, intangible capital share and intangible investment as a share of gross value-added give the same information following the assumption from Karabarbounis and Neiman (2014) that the ratio of the nominal value of the capital stock to nominal investment is constant and that the required rate of return on capital is constant (See Section IV.B of Karabarbounis and Neiman (2014)), together with the depreciation rate of intangible capital computed using BEA data consistently.

is able to reproduce a key relationship (i.e., a negative correlation between firm size and firm-level labor share documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2020)) that will affect the aggregate results on concentration and labor share, as I show later.

In particular, I do three things to match firm distribution. First, I assume that the persistent firm-specific productivity in producing a consumption/physical investment goods bundle, z, follows a non-Gaussian process. More specifically, z is drawn from a time-invariant distribution, $G(z; z^L, z^H, \gamma^z)$, which is a bounded Pareto distribution. In each period, a firm retains its previous level of productivity in producing numeraire goods with a fixed probability ρ_z, whose value is set externally. Second, I introduce permanent heterogeneity in the scale parameter η. I consider a three-point distribution with support $\{\eta_L, \eta_M, \eta_H\}$. The parameters that govern the bounded Pareto distribution $G(z; z^L, z^H, \gamma^z)$ including the upper bound z^H, the lower bound z^L, and the shape parameter γ^z as well as the parameters that govern the distribution on η are calibrated internally to match the skewed firm size distribution in terms of employment from the Business Dynamic Statistics (BDS). The largest scale parameter η_H is chosen such that $[(1 + \mu)(1 - \alpha) + \alpha]\eta_H > 1$, which means that the production technology for firms with high scale parameter η_H features increasing returns to scale. Both the non-Gaussian process of z and permanent heterogeneity in η dramatically improve the results in matching the skewed firm size distribution. See Table 3 for a comparison between the model and the data.[21] The average scale across firms is around 0.81, which is not far away from the value of the scale parameter used in the macro literature on heterogeneous firms (see, for instance, Buera and Shin (2013); Khan and Thomas (2013)). Third, I calibrate the distribution of productivity in producing intangible investment goods z_I to capture the fact that intangible-intensive firms are large on average. In the baseline case, I consider z_I to follow a two-point distribution with support $\left\{z_I^L, z_I^H\right\}$ where z_I^L is normalized to 1 to target the moment that the top 10% firms in terms of the intangible-investment-to-total assets ratio account for 51.9% of total sales in the early 1980s.[22] This is in line with Crouzet and Eberly (2019).[23] The gap between z_I^L

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[21]Since there is no way to know the number of employees of each firm directly in the model, here is how I map from my model to the data. I divide firms into five groups based on the population share from the data (this is why the model can perfectly match the data in terms of the population share) and then I can get the employment cutoff for each group of firms. Finally, I can compute the employment share of each group of firms from my model and compare the results with the data.

[22]This is a moment I document from Compustat. As I have mentioned before, I am trying to target the whole demographics of employer firms in the U.S. economy-not only the very large public firms. To fix this discrepancy, I filter Compustat firms based on the same criterion as the corporate firms. That is, after I do the simulation, I choose a cutoff for $y + px_I$, call it $\bar{v}$, such that the total income generated by all the firms with $y + px_I \geq \bar{v}$ divided by the total income generated by all the firms in the economy equals the corporate income as a share of the domestic business income from BEA-NIPA, and then I compute the relevant moments only for firms with $y + px_I \geq \bar{v}$. For measurement of intangible capital at the firm-level, see Appendix A.2

[23]They find that a firm's market share in its industry is higher when its intangible capital to total assets is higher,

and z_I as well as the corresponding support also affect the firm size distribution. In subsection 4.2, I allow z_I to take more values and compare the results with the baseline case.

The initial productivity distribution for producing numeraire goods for entrants Γ_0 is exponential with the rate parameter equal to one. The mean $\hat{z}_0$ is chosen to match the average size of start-ups relative to that of incumbent firms. I calibrate the initial tangible assets $\bar{a}_0$ to match the start-up debt-to-output ratio relative to the aggregate debt-to-output ratio. The value of $\bar{a}_0$ also affects the average size of start-ups relative to that of incumbent firms, although not as significantly as $\hat{z}_0$. Calibrating the initial conditions of entrants to match the data is important since it matters for firm distribution and the results on concentration and firm dynamics. Having start-ups too large compared to the data results in increased firm entry and declined concentration when the same technology shock that drives intangible investment is fed into the model.

For financial frictions, I calibrate the collateral parameter λ which is economy-wide to match the aggregate debt-to-value-added ratio for the private business sector.

For the remaining three parameters related to firm dynamics: exogenous exit probability π_d, entry cost κ_e, and the operating cost κ_o, I calibrate them to match three moments from the data together: the five-year survival rate, the annual entry rate, and the average exit rate of firms with fewer than 20 employees relative to that of firms with more than 500 employees. Entry cost κ_e has a more direct impact on firm entry decisions. While an increase in π_d and κ_o respectively both reduces the five-year survival rate of firms, they affect the average exit rate ratio of firms of different sizes in the opposite direction. The reason is that the exit of very large firms is driven only by the exogenous death shock π_d, while the exit of relatively small firms can be due to both the exogenous death shock π_d and operation cost κ_o. When κ_o increases relative to π_d, the ratio of the average exit rate of firms with fewer than 20 employees to that of firms with more than 500 employees increases.[24]

3.3 Cross-Sectional Implications

I now explore the main cross-sectional implications of the model at its steady-state equilibrium, calibrated to the early 1980s.

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and this relationship holds between firms of the same industry, within firms over time, and controlling for year effects. I show my model is able to reproduce their results in Table 11 of Appendix A.2.

[24]There are also technical reasons for introducing the exogenous death shock. First, it is a simple way to avoid the situation where financial frictions are irrelevant when firms survive long enough (Khan and Thomas, 2013). Second, without the exogenous death shock or if its value is very small, it is much harder to find the stationary distribution of firms.

Moment	Data	Model	Source
Employment share: $Age \leq 1$	0.036	0.038	
Employment share: $Age \in (1,10]$	0.297	0.170	BDS
Employment share: $Age \geq 11$	0.667	0.792	
Regression coefficient of payroll-to-sales on firm size (sales)	**-0.837**	**-0.692**	ADKPV
Top 10% concentration (sales)	0.675	0.724	
Persistence of intangible capital/total assets	0.780	0.752	Compustat

ADKPV: Autor, Dorn, Katz, Patterson, and Van-Reenen (2020), LBD census data -Various sectors; regression coefficient within the range $[-2.37, -0.35]$, with value-added weighted average -0.837

Table 4: Non-Targeted Moments

Non-Targeted Moments Table 4 reports some empirical moments not targeted in the calibration and their model-generated counterparts. The model can replicate fairly well the distribution of employment by firm age, which is not explicitly targeted.

I show that my model can reproduce the negative relationship between firm size and firm-level labor share, documented by Autor, Dorn, Katz, Patterson, and Van-Reenen (2020). More specifically, they regress the payroll-to-sales ratio on each firm's sales as a fraction of total sales using LBD Census data for six main sectors. They obtain a range of the estimates for the regression coefficients, $[-2.37, -0.35]$, which have a value-added-weighted average of -0.837.[25] Using the simulated data from the calibrated model,[26] I find a regression coefficient of -0.692, which is within the range and fairly close to the weighted average of the data -0.837. The share of sales going to the largest 10% of firms is also a non-targeted moment but is well captured by the model.

In addition, the persistence of the intangible capital to total assets in the model is close to its empirical counterpart, validating the introduction of the permanent idiosyncratic productivity in producing intangible investment goods.[27] If shocks on z_I become less persistent, the value of persistence generated from the model will be even smaller.

Life Cycle Implications I now explore the firm heterogeneity in productivities z and z_I as well as operating scale η in driving the life cycle patterns to further inform the results that will be

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[25]The six sectors are wholesale trade, finance, manufacturing, retail trade, utilities + transportation, and services, where wholesale trade has the most negative correlation, i.e. -2.37, and service has the least negative correlation, i.e. -0.35. The coefficient of manufacturing sector is -0.90.

[26]Here is how I map from the model to the data: since the model abstracts from intermediate goods, there is no difference between sales and value-added. Moreover, since the firm-level census data still treats intangibles such as software and R&D as expenses rather than investments, I let y to be sales in my model. That is, I regress $w\left(l_1 + l_2 + \kappa_o\right)/y$ on y/Y.

[27]See subsection 4.2 where I allow z_I following AR(1) process and the quantitative results on concentration and labor share driven by the intangible-specific technical change do not change.
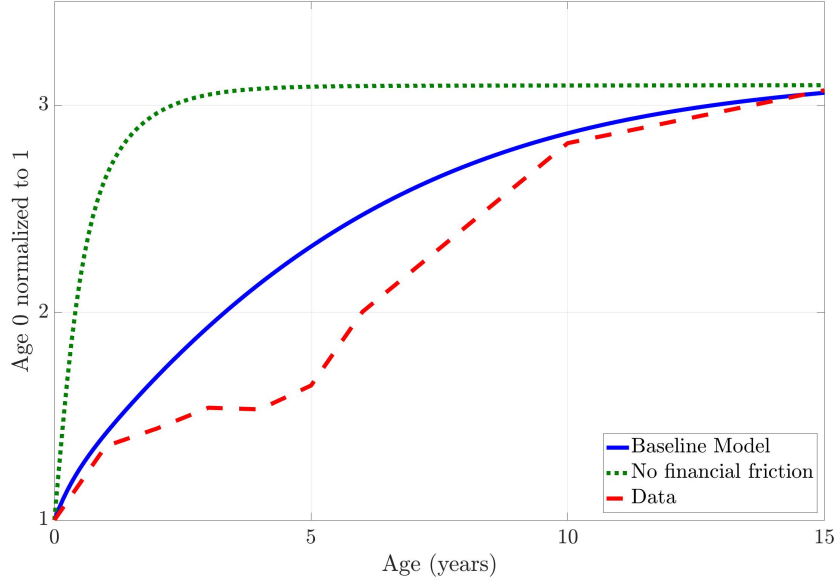
Figure 2: Average Life Cycle of Firms: Model v.s. Data

presented in Section 4. The main finding is that the labor share of firms with sufficiently large z, z_I, and η declines as those firms grow in size over the life cycle. This is consistent with Kehrig and Vincent (2021) who use establishment level data in the U.S. manufacturing sector and find that the aggregate reallocation of value added toward low-labor-share establishments is due to units whose labor share fell as they grew in size.

In Figure 2, I plot the average firm size in terms of employment over the life cycle generated from the baseline economy as well as a counterfactual economy without financial frictions (when $\lambda \to \infty$). Then, I compare them with the data. The baseline model features a realistic life cycle dynamics due to two elements: financial frictions and the costly accumulation of intangible capital. The first element plays a dominant role. Without financial frictions, firms jump to their optimal level almost immediately, which affects the aggregate results on concentration nontrivially in response to a technology shock (see subsection 4.2 for more details).

To explore the role of permanent heterogeneity in scale parameter η in the life cycle behavior of firms, I plot the average firm size in terms of employment and tangible assets, as well as firm-level labor share and intangible capital-to-physical capital ratio for firms with different scale parameter η from birth to maturity, as in Figure 3. Panels A and B show that η_H firms, those whose technology features increasing returns to scale due to a sufficiently large value of η and the non-rivalry property of intangible capital, account for the upper tail in the size and growth rate distributions.
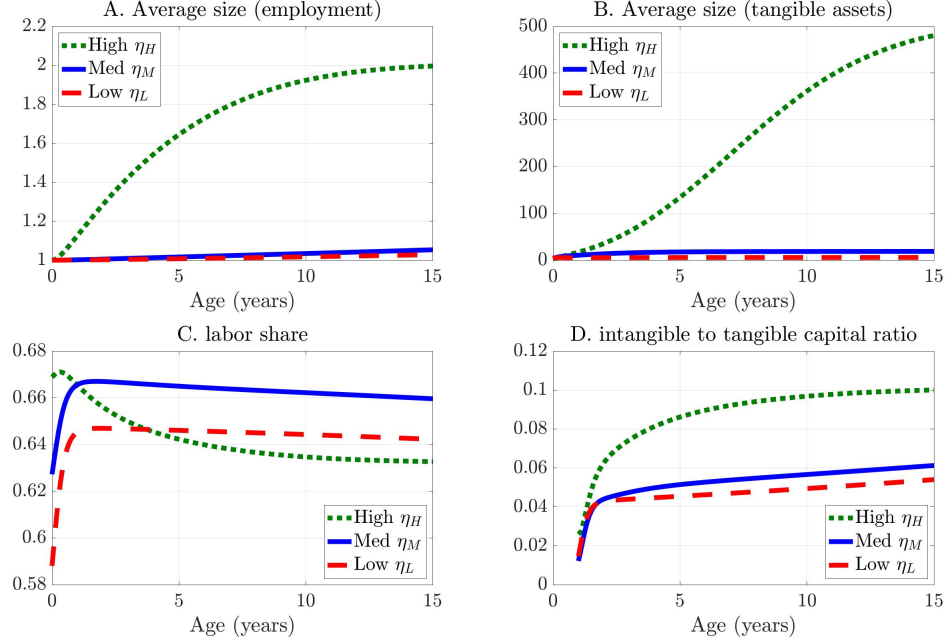
26

Figure 3: Average Life Cycle of Firms: firms with different scale parameters η

More importantly, I check how heterogeneity in η affects the firm-level labor share over the life cycle. The firm-level labor share is defined as $w(l_1 + l_2 + \psi_o)/(y + px_I)$. Initially, when firms have little intangible capital, η_H firms have the highest labor share, which is consistent with the income share results of a standard Cobb-Douglas technology. However, as firms accumulate more intangible capital in-house, η_H firms produce more efficiently and use less and less labor relative to the value-added. This is, again, due to the sufficiently large value of η and the non-rivalry property of intangible capital, which allows η_H firms to operate at increasing returns to scale as they accumulate more and more intangible capital. Panel D shows that firms accumulate more intangible capital relative to physical capital as they age. Since all firms start with zero intangible capital and are thus unable to produce and no need to rent physical capital, the intangible capital-to-physical capital ratio is not defined at age zero. Among all firms, η_H firms' intangible-to-physical capital ratio is the highest.

In Figure 4, I plot the average firm size in terms of employment and tangible assets, as well as the firm-level labor share and intangible capital-to-physical capital ratio for firms with different productivity levels in producing intangible investment goods from birth to maturity. Panels A and B show that firms with high z_I, those with (permanent) high productivity in producing intangibles, account for the upper tail in the size and growth rate distributions. These firms take more advantage of an IISTC. Together with Panels C and D, these figures imply that firms with higher z_I are more
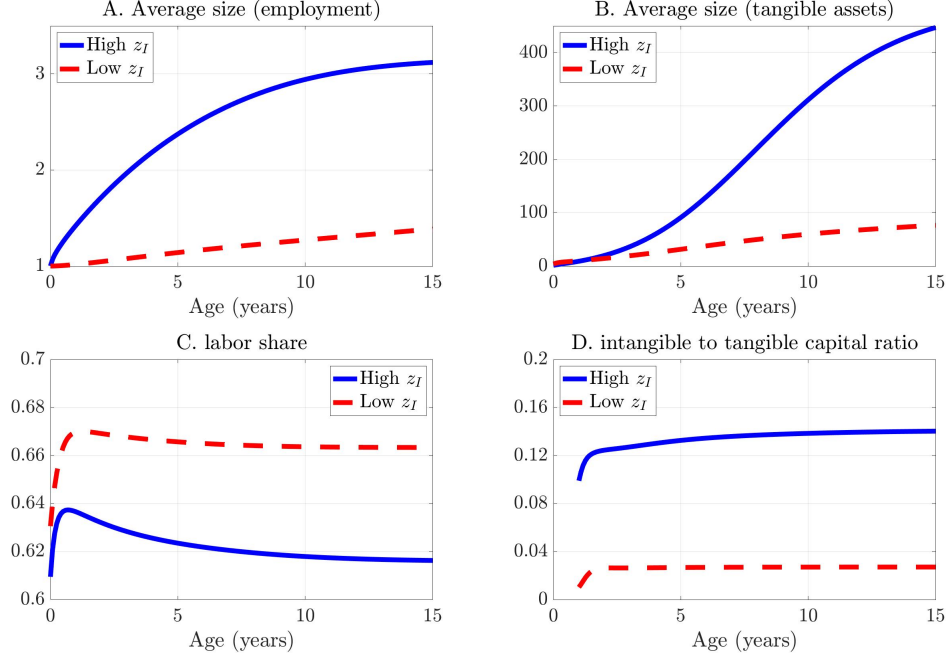
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Figure 4: Average Life Cycle of Firms: high z_I firms v.s. low z_I firms

intangible capital intensive, less labor intensive, and larger. The growth patterns for firms with different intangible capital intensities are also in line with Crouzet and Eberly (2019), who find that firms with higher intangible capital intensities are larger and grow faster. The reason the intangible-to-tangible capital ratio starts to have value since age one is, again, that all firms start with zero intangible capital and are thus unable to produce and have no need to rent physical capital at age zero.

How does the difference in the productivity in producing the bundle of consumption/physical investment goods z affects firms' behavior over the life cycle? As shown in Figure 5, it is not surprising that, given other idiosyncratic characteristics, firms with higher z are larger and grow faster (Panels A and B). Additionally, high z firms have a lower intangible-to-physical-capital ratio. This is because it is more costly for high z firms to produce intangible investment goods, conditioned on other idiosyncratic states. Although the intangible-to-tangible ratio seems to be even slightly lower as high-z firms age, those firms still accumulate much more intangible capital over the life cycle than firms with lower z, as shown in Panel B.

What is surprising in Figure 5 is that, in terms of firm-level labor share, firms with high z only use slightly higher labor input relative to the value-added, compared to those firms with z in the middle of the distribution. This is driven by two effects that work in the opposite direction. On
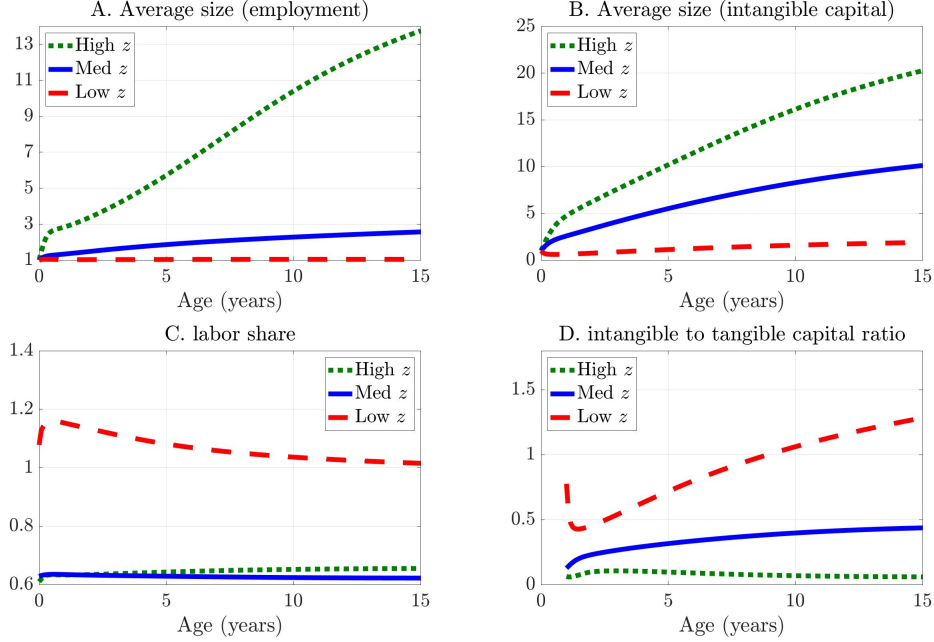
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Figure 5: Average Life Cycle of Firms: firms with different productivity z

the one hand, high-z firms, conditioned on other firm-specific states, find it more costly to produce intangible investment goods. This leads to a higher labor share. On the other hand, high-z firms also invest much more in intangible capital than other firms, as suggested by Panel B. Due to the non-rivalry property of intangible capital, their labor share is thus lower. The consequence of the two opposing effects is that high-z firms' labor share is only slightly higher than that of middle-z firms. The reason why firms with lowest z have a very high labor share is that due to very low productivity z, they invest little in intangible capital, as suggested by Panel B, which makes them unable to utilize the non-rivalry property of intangible capital, but they still need to pay overhead labor cost if they choose not to exit the market. Consequently, low z firms have very high labor shares.

3.4 Discussion

Before moving forward, I summarize the key elements of the model that matter for the qualitative results as well as the deep parameters and their corresponding calibration strategy that matter for the quantitative results. In the model, I utilize two distinct economic properties of intangible capital. First, intangible capital is firm-specific in the sense that each individual firm accumulates its own intangible capital by producing intangible investment goods in-house. This helps generate hetero-

geneous income shares across firms. Second, the usage of intangible capital is non-rival in the sense that intangible capital can be used to simultaneously produce both numeraire goods and intangible investment goods rather than being split between them. This also helps generate heterogeneous income shares across firms by breaking up the constant factor share result with Cobb-Douglas technology. More importantly, with positive intangible capital share μ and a sufficiently large scale parameter η, the non-rivalry property of intangible capital allows the production technology to feature increasing returns to scale. This leads to the result that firms with high z_I, sufficiently large z and η are large and have low labor shares. These two elements enable the model to generate a negative correlation between firm size and firm-level labor regardless of the calibration.

I now highlight two sets of deep parameters that drive the quantitative results. The first one consists of parameters that govern the permanent heterogeneity in scale parameter η, which are calibrated to match the skewed size distribution of firms to ensure that large firms are sufficiently large. The second set governs the distribution of firm-specific permanent productivity for intangibles z_I, which is calibrated to capture the empirical fact that intangible-intensive firms are large. Disciplining these key parameters in a proper way ensures that the negative correlation between firm size and firm-level labor share, as shown in Table 4 , is strong enough to generate the main results of the paper.

4   Main Results

This section uses the calibrated model from Section 3 to study the aggregate implications of an intangible-investment-specific technical change (IISTC). The main results are summarized in subsection 4.1 where I quantify the long-term impact of the IISTC on the measured labor share and concentration. In subsection 4.2, I investigate the key mechanism that drives the quantitative results.

4.1   Aggregate Implications

IISTC is modeled as a permanent increase in the aggregate productivity in producing intangible investment goods A_I to match the decline in the relative price of intangible investment goods from the BEA data, which is constructed as the ratio of the price of investment in intangible capital including software, R&D, and artistic originals to the price of the bundle of consumption and investment in traditional physical capital including structures and equipment. For 1980-2016, the relative price of

	Start of sample		End of sample		Change (pp.)	
	Model	**Data**	**Model**	**Data**	**Model**	**Data**
Labor share $\frac{wL}{Y+PX_I}$ (post-2013 revision)	0.640	0.640	0.617	0.594	-2.3	-4.6
Labor share $\frac{wL}{Y}$ (pre-1999 revision)	0.667	0.660	0.657	0.631	-1.0	-2.9

Start of the sample: 1980-1985 average (1975-2016 linear trend)
End of the sample: 2011-2016 average (1975-2016 linear trend)

Table 5: Aggregate Implications: Measured Labor Share

intangible investment goods declines by 44%.[28]

To be consistent with how BEA constructs the price indices, the aggregate relative price of intangible investment goods in the new steady state relative to its value in the initial steady state is given by the Fisher formula:

$$P_{new,initial} = \sqrt{\frac{\int p_{new} x_{I,initial} d\varphi_{initial}^p}{\int p_{initial} x_{I,initial} d\varphi_{initial}^p} \times \frac{\int p_{new} x_{I,new} d\varphi_{new}^p}{\int p_{initial} x_{I,new} d\varphi_{new}^p}}$$

where p is the shadow price of the intangible investment goods relative to the numeraire, as in equation 3. Then the percentage change in the aggregate relative price of intangible investment goods from the initial steady state to the new steady state is $\%\Delta = (P_{new,initial} - 1) \times 100$. In the model, I increase the aggregate productivity in producing intangible investment goods A_I by 79% to match the 44% of the observed decline in the relative price from the data.

I report the main results in Table 5 and Table 6. The start of the sample I am targeting is the early 1980s and the end of the sample is the 2010s. For measured labor share, the data I am targeting is the year 1980 (initial steady state) versus the year 2016 (new steady state after the technical change has occurred) in the linear trend of the BEA-measured labor share for 1975-2016.[29] I consider two measures of labor share. The first measure (post-2013 revision) is after BEA-NIPA capitalizes both software and R&D. The second one (pre-1999 revision) is before BEA starts to treat software and R&D as final output. Based on the data, the magnitude of the decline in the labor share of the post-2013 revision of BEA is larger than that of the pre-1999 revision. This is consistent with the increased intangible investment as a share of gross value-added, which implies

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[28]For more details about the construction of the relative price of intangible investment, see Appendix A.

[29]Choosing 1975 as the starting year is to ensure better estimates and consistency for intangible investment because 1975 is the first year that the Federal Accounting Standards Board (FASB) required firms to report R&D, and hence, the measured labor share.

	Start of sample		End of sample		Change (pp.)	
	Model	Data	Model	Data	Model (pp.)	Data (pp.)
Annual Firm Entry rate	13.6	12.5	12.1	8.0	-1.5	-4.5
Employment Share: Large Firms (500+)	44.1	47.0	48.5	51.7	+4.4	+4.7
Employment Share: Mature Firms (11 years +)	79.2	66.7	85.2	80.5	+6.0	+13.7
Top 10% concentration (Sales)	72.4	67.5	76.9	72.8	+4.5	+5.3

Start of the sample: 1980-1985 average; End of the sample: 2011-2016 average

Table 6: Aggregate Implications: Concentration

that the drop in the aggregate labor income share would be smaller if intangibles are not treated as final output.[30] As shown in Table 5, the IISTC, or the fall in the relative price of intangible investment goods, is able to explain approximately 50% of the decline in the BEA-measured labor share of both the post-2013 revision and the pre-1999 revision.

The decline in the pre-1999 revision BEA-measured labor share is purely driven by the real location effect: due to the negative correlation between firm size and firm-level labor share, the IISTC shifts firm distribution toward large and more intangible capital-intensive firms with low labor shares. The decline in the post-2013-revision-BEA-measured labor share is due to both the reallocation effect and the measurement issue: intangibles that are treated as final output have increased relative to other components of final output during the past three decades.

For the statistics related to concentration, the data I am focusing on are the average of 1980-1985 (initial steady state) versus the average of 2011-2016 (new steady state with the technical change). For concentration, I look at three measures: (1) the employment share of large firms with more than 500 employees; (2) the employment share of old firms with more than 11 years of operation; and (3) the share of sales going to the largest 10% of firms.[31] As shown in Table 6, the IISTC, targeted to match the decline in the relative price of intangible investment goods, accounts for around 1/3 of the decline in the annual firm entry rate, 93.6% of the increase in the employment share of large firms, slightly less than half of the increase in the employment share of mature firms, and around 84.9% of the rise in the market concentration.

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[30]For a more detailed discussion on different measures of labor share, see Appendix B.

[31]The data on the first two measures comes from BDS. The data on concentration in terms of sales comes from Autor et al. (2020) for various sectors weighted by industry sales shares, and the empirical moment is the average of the years 1987-1992.

Aggregate Elasticity of Substitution Finally, I numerically derive the aggregate elasticity of substitution. Given that the distribution of firms shifts toward more productive and more intangible capital-intensive firms with lower labor shares, I want to explicitly see if the aggregation of individual firms mimics an aggregate CES production technology with an elasticity of substitution among factors greater than one in response to an IISTC. [32]

Since there is no specific form of the aggregate production function in my model, I compute the aggregate elasticity of substitution between factors in production as follows. First, I compute the aggregates of factor inputs—physical capital stock K_T, intangible capital stock K_I, and labor N as well as the factor prices R_T, R_I, w—for the three types of inputs respectively at both the initial steady state and the new steady state with the IISTC fed into the model. Then, the aggregate elasticity of substitution between K_T and K_I is computed using a midpoint rule as follows:

$$
\sigma_{K_I K_T} = \frac{dln\left(\frac{K_I}{K_T}\right)}{dln\left(\frac{R_T}{R_I}\right)} \underset{midpoint\,rule}{\cong} \frac{\frac{\left(\frac{K_I}{K_T}\right)_{final} - \left(\frac{K_I}{K_T}\right)_{initial}}{\frac{1}{2}\left[\left(\frac{K_I}{K_T}\right)_{final} + \left(\frac{K_I}{K_T}\right)_{initial}\right]}}{\frac{\left(\frac{R_T}{R_I}\right)_{final} - \left(\frac{R_T}{R_I}\right)_{initial}}{\frac{1}{2}\left[\left(\frac{R_T}{R_I}\right)_{final} + \left(\frac{R_T}{R_I}\right)_{initial}\right]}}
$$

The aggregate elasticity of substitution between intangible capital K_I, and labor N , $\sigma_{K_I,N}$, and between physical capital K_T, and labor N , $\sigma_{K_T,N}$ are computed in the same manner.

I aggregate intangible capital as $K_I = \int k_I d\varphi^p$ which implicitly assumes that k_I produced by individual firms are perfect substitutes. This is consistent with how I aggregate intangible investment goods x_I for each individual firm in subsection 4.1. Physical capital used to produce consumption/physical investment goods y is aggregated as $K_{T1} = \int k_{T1} d\varphi^p$. Physical capital used to produce intangible investment goods x_I is aggregated as $K_{T2} = \int k_{T2} d\varphi^p$. Then, aggregate physical capital is $K_T = K_{T1} + K_{T2}$. I assume the physical capital rental rate is $R_T = \int (r + \delta_T + \zeta) d\varphi^p$, where ζ is the Lagrange multiplier of the collateral constraint faced by each individual firm, which is a nonnegative number.[33] Finally, the model's equilibrium aggregate cost of intangible capital is

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[32]Consider a simple example where there are only two perfectly competitive firms: one labor-intensive firm with production technology $Y_1 = L$ and the other is a capital-intensive firm with production technology $Y_2 = AK$. There are constant labor shares within each firm. Suppose there is capital-specific technical change such that A increases. Under certain conditions, as capital becomes relatively cheaper, it is substituted for labor on the aggregate-level. For more details, see Karabarbounis (2018).

[33]Note that physical capital k_{T1} used to produce y for each individual firm is generated by solving a *static* profit maximization problem, so the aggregate cost of physical capital k_{T1} used to produce y is $R_{T1} = \int (r + \delta_T + \zeta) d\varphi^p$. Physical capital k_{T2} used to produce x_I for each individual firm is generated after the choice of intangible capital k_I' for next period is made. More specifically, given k_I', I can solve for x_I, which is equal to $k_I' - (1 - \delta_I) k_I$. Based on production technology for x_I (equation (2)), I have $k_T^{(1-\mu)(1-\alpha)\eta} l_2^{\alpha\eta} = \frac{k_I' - (1-\delta_I)k_I}{A_I z_I k_I^{\mu(1-\alpha)\eta}}$. Then I get k_T, l_2 separately by

defined by:

$$R_I = \frac{\int y\,d\varphi^p + \int px_I\,d\varphi^p - W\bar{N} - R_T K_T}{K_I}$$

Admittedly, defining R_I in this way means $R_I K_I$ not only include intangible capital rents, i.e., profit generated from K_I, but also profit from some firms with scale parameter $\eta < 1$ because the current framework cannot separate the two.

The results are

$$\sigma_{K_I, K_T} = 1.589, \sigma_{K_I, N} = 1.453, \sigma_{K_T, N} = 0.359$$

We can see that the aggregate elasticity of substitution between intangible capital K_I and physical capital K_T as well as between intangible capital K_I and labor N in production both exceed one, with σ_{K_I, K_N} being slightly larger. This is because R_I drops driven by the decline in the relative price of intangible investment goods, while R_T slightly increases since more firms have binding collateral constraints after the technical change, increasing the effective rental rates of physical capital, and the increase in w is larger than the increase in R_T. The aggregate elasticity of substitution between physical capital K_T and labor N is much smaller and less than one. Therefore, the aggregation of heterogeneous firms' production mimics the behavior of an aggregate CES production function with elasticity of substitution between capital and labor greater than one, as in Karabarbounis and Neiman (2014).

4.2 Alternative Setups

To identify the most essential elements of the baseline model and the deep parameters that drive the key relationships that generate the results on concentration and labor share led by the IISTC, I consider a number of alternative setups. First, I discuss the role of heterogeneity in the scale parameter η played in generating the quantitative results of the baseline model. The cases I am considering are: (1) a standard Gaussian process in z without heterogeneity in η; (2) a standard Gaussian process in z with heterogeneity in η; and (3) no heterogeneity in η. Second, I consider some other setups that can illustrate the importance of other elements aside from heterogeneity in η in the baseline model, which include: (1) no heterogeneity in z_I; (2) persistent z_I shocks (AR1); (3) no overhead labor; (4) no financial friction ($\lambda \to \infty$); (5) making intangible capital k_I as collateralizable as tangible assets; and (6) allowing equity issuance at a cost. Finally, I consider an alternative model where firms operate standard Cobb-Douglas technology and accumulate intangible

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solving a maximization problem given factor prices $R_T = \int (r + \delta_T + \zeta)\, d\varphi^p$ and w.

capital at a price that is common across firms rather than within firms. I recalibrate parameters in all of these experiments to reproduce the same set of moments in the data as I have done in the baseline economy.

I list the results in Table 7. In this table, I first report three cross-sectional moments in the first column that inform the macro moments in the second column. The first two cross-sectional moments are (1) the employment share of very large firms (500+ employees) and (2) the market share of the top 10% firms in terms of the intangible investment-to-total assets ratio. These two moments have direct impacts on the third cross-sectional moment: the negative correlation between firm size and firm-level labor share. The ability to capture this key relationship well in the model is essential to the main results on concentration and labor share. For each alternative setup considered in the table, I check how each element of my baseline model helps to generate the three cross-sectional moments, thus contributing to the macro results on concentration and labor share driven by the IISTC.

The Role of Heterogeneity in η Recall that the benchmark model relies on two elements to match the skewed firm size distribution: (1) the non-Gaussian process for the productivity in producing numeraire goods z and (2) heterogeneity in scale parameter η. To explore the role of heterogeneity in η, I first assume z follows a standard Gaussian process. That is, z follows an $AR(1)$ process in logs: $logz^{'} = \rho_z logz + \varepsilon^{'}$, with $\varepsilon^{'} \sim N(0, \sigma_z)$, with heterogeneity in η as in the baseline model. Without the non-Gaussian setup, the cross-sectional moments in the employment share of large firms and the market share of the most intangible-intensive firms can still be matched well, although not as well as in the baseline. Consequently, the negative correlation between firm size and firm-level labor share is slightly weakened compared to the baseline, which directly affects the magnitudes of both the increase in concentration and the decline in labor share.

The second case I consider is that z still follows the non-Gaussian process as in the baseline but there is no heterogeneity in η. Although the three cross-sectional moments can be matched well, the magnitudes of both the increase in concentration and the decline in the labor share are much smaller compared to the baseline. The reason is as follows. Without heterogeneity in η, we need the upper bound of z in the bounded Pareto distribution to be very high to match the employment share and market share of large firms. However, this simultaneously drives the entry rate, and thus the exit rate, as well as the employment share of small firms, down to be very low. To match these moments as in the benchmark, we need a very high operation cost κ_o (which is denominated in labor units).

In this case, $\kappa_o = 1.0$, which is almost ten times the baseline calibrated value. As a result, overhead labor plays a much more important role in driving the labor reallocation from small firms to large firms. In other words, this alternative setup becomes less sensitive in response to the IISTC shock compared to the baseline.

The third case is that neither the non-Gaussian process of z nor the heterogeneity in η is assumed. Without these, the first two cross-sectional moments cannot be well matched, thus weakening the negative correlation between labor share and firm size. The direct consequence is that the magnitudes of both the increase in concentration and the decline in labor share are much smaller compared to the baseline.

In summary, the heterogeneity in the scale parameter η is a key element of the benchmark model. Due to the firm specificity and the non-rivalry property of intangible capital, a high η does not necessarily mean a high labor share at the firm-level (as in a standard Cobb-Douglas technology). Based on the quantitative results of the paper, firms with higher η, in general, are larger, more intangible-intensive, and have a low labor share.

Other Model Elements   The next two cases concern the productivity in producing intangible investment goods z_I. The heterogeneity in z_I is important to matching the fact that more intangible-intensive firms (in terms of intangible investment to total assets) are larger (in terms of market share), thus contributing significantly to the results on the declined labor share. Introducing permanent shocks to z_I also helps to match the skewed firm size distribution, thus contributing to the results on the rise of concentration.

In the baseline economy, productivity in producing intangible investment goods z_I is a permanent shock. Modeling it as a persistent shock following an AR(1) process and introducing more grids improve the results, but to a very limited extent.[34]

Overhead labor contributes to better results as well, to a limited extent, for concentration in terms of sales and employment. However, it improves the results on firm entry significantly because the cost of the technological advances (i.e., the increased equilibrium wage) dominates the benefit of them for start-up firms when calibrating their initial conditions (i.e., wealth and productivity) to match the data.

The last three cases in this section are about financial frictions. In the first one, I let the collateral

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[34]I increase the number of grids of z_I from two (in the baseline) to five, and calibrate the persistence of the productivity process ρ_{z_I} as well as the standard deviation of productivity shocks σ_{z_I} to target the persistence of the intangible investment to total assets ratio and the standard deviation of the intangible investment to total assets ratio respectively.

parameter λ go to infinity so that there is no financial friction. In that case, the moments on employment share of large firms and market share of the top 10% in terms of intangible-investment-to-total-assets ratio overshoot the data. As we can see from Figure 2, this setup does not feature a realistic life-cycle of firms in the sense that firms jump to their optimal level almost immediately after their birth compared to the baseline. Consequently, the moments on concentration are very sensitive to the value of technology parameters. An improved aggregate technology for producing intangibles disciplined by the decline in the relative price of intangible investment goods drastically increases the concentration in terms of sales and employment. The increase in the model, in percentage points, is almost six times as large as that in the data. Moreover, it drives an increase in the annual entry rate of 11 percentage points, while its empirical counterpart declines by 4.5 percentage points. These results imply that having a model with a realistic life cycle of firms is important in studying concentration and firm dynamics.[35] In the second case, intangible capital k_I is made to be equally collateralizable as tangible assets so that the collateral constraint becomes $k_T \leq \lambda a + (\lambda - 1) p k_I$.[36] Such financial friction also contributes to a realistic life cycle of firms so there is almost no change in the results on concentration compared to the baseline case. This implies the non-collateralizability assumption of intangible capital is not crucial to explain the change in firm concentration (or labor share).[37] Since in my model, intangible capital is accumulated within firms and cannot be traded, assuming that intangible capital has no collateral value seems more natural. I choose my baseline setup also to be consistent with the empirical evidence as well as the existing literature that studies implications of a lack of collateral value for intangible capital (see Ai et al. (2019); Caggese and Perez-Orive (2018); Chen (2014); Falato et al. (2018); Garcia-Macia (2017); Wang (2017)). In the third case, I allow a portion of the firms (corresponding to public firms) to issue equity at a cost: $d + \boldsymbol{H}(d) \boldsymbol{1}_{\{\mathbf{d} \leq \mathbf{0}\}}$ where $\boldsymbol{H}(d) = -\iota|d|$, and ι is chosen to match the equity-to-asset ratio in Compustat, while the remaining firms continue to face nonnegative payment conditions. There is almost no change in the quantitative results.

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[35] In Aghion et al. (2019), the concentration in terms of sales share of top 10% firms increases by 35.1 percentage points (while the data is 5.3 p.p.) driven by fallen firm-level costs of spanning multiple markets matched to the change in between component of labor share from the data

[36] This is derived from: $b' \leq \theta \left(k_T' + p k_I' \right)$ where $b := k_T - a$, plus the timing assumption following Moll (2014) and Midrigan and Xu (2014).

[37] However, if the focus of the paper is to explain the debt-financing patterns of firms with different intangible intensity, firms' cash holdings, or the cyclicality of equity returns, this assumption will be the key to the results. See, for example, Wang (2017), Falato et al. (2018), and Ai et al. (2019). Consistent with these papers, the baseline model featuring the assumption that intangible capital cannot be used as collateral contributes to explaining the rise of corporate saving flows driven by the intangible-investment specific technical change since the early 1990s, but this is not going to be the focus on of my paper. The rise of corporate saving is a byproduct of the decline in labor share. See Chen, Karabarbounis, and Neiman (2017).

	Cross-sectional Moment			Macro Moment				
Cases	Emp. share large firms (500+)	Market share top 10% firms (intan. invest.)	Regression coefficient labor share on firm size	$\triangle$ Concentration			$\triangle$ Labor share	
				emp.	sales	entry	old	new
Data	47.0	51.9	-0.84	4.7	5.3	-4.5	-2.9	-4.6
Baseline	44.1	46.3	-0.69	4.4	4.6	-1.5	-1.0	-2.3
The role of heterogeneity in η								
$z \sim AR(1)$ with heterog. η	39.8	42.1	-0.62	3.9	4.1	-1.1	-0.9	-2.1
$z \sim Pareto$ no heterog. η	40.2	43.9	-0.64	0.4	0.3	-0.1	-0.2	-0.4
$z \sim AR(1)$ no heterog. η	13.2	35.0	-0.24	0.0	0.1	-0.0	-0.2	-0.5
Other model elements								
No heterog. z_I	35.9	35.1	-0.46	0.9	1.3	-0.1	-0.6	-1.4
z_I persistent	44.3	47.4	-0.69	4.4	4.5	-1.5	-1.0	-2.3
No overhead	44.0	46.1	-0.60	3.9	4.1	0.0	-0.9	-2.2
$\lambda \to \infty$	58.1	75.9	-1.31	30.1	29.6	11.2	-1.2	-2.4
k_I collateral	44.3	46.3	-0.69	4.2	4.5	-1.5	-1.0	-2.3
Equity issu.	44.1	46.3	-0.69	4.4	4.6	-1.5	-1.0	-2.3
Standard Cobb-Douglas technology								
Full model	43.2	9.8	-0.42	-0.2	0.4	-1.1	2.2	1.2
No overhead	43.2	9.4	2.58	1.3	0.4	2.1	4.8	3.9
Neither heterog. η nor overhead	42.3	9.2	0.00	0.1	0.3	0.7	0.9	0

Table 7: Alternative Setups

Standard Cobb-Douglas Technology   The two most important features of the baseline economy are (1) firm-specific intangible capital and (2) the non-rivalry property of intangible capital. Both assumptions (1) and (2) are necessary to generate heterogeneous income shares across firms. Assumption (2) is key to the result that very large firms feature increasing-returns-to-scale technology and that large firms, in general, use more intangible capital relative to labor and have a low labor share. In this alternative framework, there is a common market for intangible capital to be traded so that each firm faces the same price of intangible capital. Furthermore, I relax the assumption of the non-rivalry property of intangible capital by making firms operate with a standard Cobb-Douglas technology. The recursive problem of incumbent firms now becomes as follows:

$$v\left(k_I, a, z, \eta\right) = \max_{k_I', a', l, k_T, d} \left\{ d + \beta \mathbb{E}\left[\frac{U'\left(C'\right)}{U'(C)} v^0\left(k_I', a', z', \eta\right) \mid z \right] \right\} \tag{16}$$

s.t.

$$\underbrace{d}_{dividend} + P_I\underbrace{\left(k_I' - (1-\delta_I) k_I\right)}_{intangible\ invest.} + a' = \underbrace{y}_{NIPA\ income} - \underbrace{wl}_{wage} - \underbrace{(r+\delta_T)k_T}_{rental\ cost} + (1+r)\, a$$

$$y = Az\left[\left(k_T^{(1-\mu)} k_I^\mu\right)^{1-\alpha} l^\alpha\right]^\eta$$

$$k_T \leq \lambda a, d \geq 0$$

where P_I denotes the price of intangible investment relative to the numeraire. Note that the differences between this model setup, which I call it the "standard Cobb-Douglas technology" case, and the baseline model are as follows. In the "standard Cobb-Douglas technology" case, (i) there is only one type of technology to produce one type of good y that can be used for investment in both physical capital and intangible capital as well as for consumption, which means that the non-rivalry property of intangible capital as well as the consequent increasing-returns-to-scale technology no longer exist; and (ii) there is no heterogeneity in productivity in producing intangibles, and the price of intangible investment is no longer firm-specific; instead, all firms face the same relative price of intangible investment, P_I.

Correspondingly, the aggregate labor share (post-2013 revision) when intangibles are treated as final output is defined as

$$S_{N,alter.} = \frac{w\bar{N}}{Y} = \frac{w\left[\int_S (l + \kappa_o)\, d\varphi^p + \int_S \kappa_e d\varphi^e\right]}{\int y d\varphi^p} \tag{17}$$

and the aggregate labor share (pre-1999 revision) is defined as

$$S_{N,pre,alter.} = \frac{w\bar{N}}{Y - P_I X_I} = \frac{w\left[\int_S (l + \kappa_o)\, d\varphi^p + \int_S \kappa_e d\varphi^e\right]}{\int y d\varphi^p - P_I \int x_I d\varphi^p} \qquad (18)$$

At the initial steady state, $P_I = 1$. I then decrease P_I to match the decline in the relative price of intangible investment goods from the data. I report the results in the last section of Table 7 under "standard Cobb-Douglas technology" and label the model defined in equation (16) as the "full model", which includes both the elements of heterogeneity in scale parameter η and the overhead labor, as the baseline model does. In this case, the negative correlation between firm size and firm-level labor share generated from the model purely relies on the introduction of overhead labor. That is, given η, a larger firm (due to the higher z) has a lower labor share because of the fixed cost κ_o, denominated in labor units. Even though the model is still able to generate the negative correlation between firm size and firm-level labor share, the aggregate labor share of both measures increase. The reasons are that a decrease in P_I shifts the distribution of firms toward firms with high η, which have a high labor share, and this effect dominates the effect whereby a larger firm has a lower labor share. This can be seen by comparing the case "Full model" with the case "No overhead": when there is no overhead labor, the correlation between firm size and firm-level labor share becomes positive, and the aggregate labor share increases even more. The old measure of labor share increases more, since intangibles are not treated as final output, as shown in equation (18), and the rising intangibles drive Y up.

Moreover, under the standard Cobb-Douglas technology case, intangible capital no longer has the non-rivalry property, and thus cannot be guaranteed that intangible-intensive firms are also large ones. Consequently, the concentration results are weakened. When there is neither permanent heterogeneity in η nor overhead labor, the negative correlation between labor share and firm size disappears, and the aggregate labor share of the new measure does not change, since firms have constant income shares regardless of the firm-level heterogeneity, while the aggregate labor share of the old measure increases by 0.9 percentage points since, again, rising intangibles are not treated as final output, which decreases the value of the denominator of the old labor share measure.

In summary, the two key features of intangible capital highlighted in the baseline model—that intangible capital is firm-specific and nonrival—are the most important elements in generating the main results. Parameters contributing to matching the skewed size distribution of firms, especially the heterogeneity in η, and the empirical fact that intangible-intensive firms are large matter for

the quantitative results because they closely relate to whether the negative correlation between firm size and firm-level labor share is strong enough. In other words, without the two key features of intangible capital, even if the model still features the deep parameters, it would not generate the main results of the paper even in a qualitative manner.

5 Policy Implications

Thus far, I have shown that IISTC simultaneously accounts for a large fraction of the decline in measured labor share and the rise in concentration. Does the IISTC, by emphasizing the greater importance of intangible capital in firms' production, change how policymakers view its impact on investment, firm creation, and the overall dynamism of the U.S. economy?

A number of researchers (e.g. Dottling, Gutierrez, and Philippon (2017); Gutierrez and Philippon (2018)) have considered the rise in concentration and the decline in firm creation to be a negative outcome, due to, for example, the weakened anti-trust enforcement, which may need to be addressed by policies such as breaking up mega firms. While the results of my model imply that the decline in the aggregate labor share and the rise in concentration is an efficient equilibrium outcome of technological advances, there are still policy implications worth discussing. For example, R&D, as one of the most important types of intangible investment, is widely acknowledged to be a significant contributor to innovation, job creation, productivity growth and welfare. Considering that the largest proportion of R&D in the United States is funded by private industry, the government creates policies that stimulate R&D activity, especially in-house R&D work, in the private sector.

In the following, I perform a simple numerical exercise to evaluate an industrial policy—an R&D investment tax credit—aimed at boosting R&D investment, using the benchmark model. This policy is prevalent and varies in different forms in many countries.[38] The main economic justification for this policy is that private firms may fail to conduct sufficient quantities of R&D investment, as it has some characteristics of a public good. In my model, there is no externality. Instead, intangible capital cannot be used as collateral and it is this financial friction that creates distortions. The R&D investment tax credit, intuitively, can change the relative price of intangible capital to physical capital and labor and boost intangible investment, when intangible capital is at a disadvantage in the debt market collateralization, and thus subject to underinvestment. This insight is in line with Wang (2017).

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[38]See, for example, Bloom, Griffith, and Van-Reenen (2002); Hall and Van-Reenen (2000), for OECD countries

I assume a tax credit with a rate of 30% imposed on the current expenditures for R&D activities. This magnitude of tax credit is in the range of empirically estimated values (Bloom, Griffith, and Van-Reenen, 2002; Hall and Van-Reenen, 2000). Due to my model setup, the tax credit is imposed on labor that is used to produce intangible investment goods.[39] I assume the tax expenditure in the model is financed by a lump-sum income tax to isolate the unintended effects of the tax credit through general equilibrium price adjustments. Note that R&D is only one of the three types of intangible investment considered in this paper, and based on the BEA-Fixed Assets Tables, it represents approximately 61% of the total intangible investment in the early 1980s. Hence, the rate of tax credit given to the total intangible investment for each firm is $30\% \times 61\% = 18.3\%$.

I consider two cases. In Case I, all the operating firms in the economy are eligible for the tax credit, while in Case II, only small and medium-sized enterprises (SMEs), defined as those hiring fewer than 500 employees, which is consistent with the definition of SMEs used in Fort et al. (2013), are targeted. Nowadays, many countries (or regions) provide SMEs with more generous tax credits or subsidies for conducting R&D activities, especially in-house R&D, for example, Canada, Hong Kong, Japan, Korea, and the United Kingdom. Since the goal of the policy evaluated here is to promote (in-house) R&D investment, to make the two cases comparable, I adjust the rate of the tax credit in Case II such that the intangible investment as a share of value-added is increased at the same magnitude as in Case I.

The budget constraint for the incumbent firm's problem after the policy is implemented becomes:

$$d + a^{'} = y - (r + \delta_T)k_T - wl_1 - (1 - \tau)\,wl_2 + (1 + r)\,a - w\kappa_o \tag{19}$$

where τ will depend on the size of the firms in Case II.

Under the policy scheme described above, I run the following experiment. I start as if the economy is in a stationary equilibrium conditional on the parameter values of the early 1980s, and I compare this economy to a new stationary economy in which an R&D tax credit is given as in equation 19.

Columns 1 and 2 of Table 8 compare a consumption equivalent measure of welfare,[40] and intangible investment, which are normalized to 1 at the initial steady state, as well as firm annual

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[39]This is consistent with reality. Countries such as Belgium and Netherlands tie R&D tax credits explicitly to jobs with a payroll withholding tax credit for R&D wages

[40]Since the model set-up assumes linear utility in consumption and perfectly inelastic labor supply in the representative household's problem, the change in the consumption equivalent measure of welfare is captured by change in the aggregate consumption.

	(1)	(2)	(3)	(4)	(5)
		R&D Tax Credit			
	Baseline	**Targeting all firms** **(Case I)**		**Targeting SMEs only** **(Case II)**	
		Level	△	**Level**	△
Intangible Investment/Value-added	0.032	0.038	18.75%	0.038	18.75%
Rate of R&D Tax Credit	-	0.183	-	0.352	-
Relative Price of Intangibles	1.000	0.918	-8.82%	0.864	-13.59%
Consumption	1.000	1.043	4.30%	1.014	-1.40%
Output (numeraire goods)	1.000	1.034	3.40%	1.024	-2.40%
Intangible Investment	1.000	1.165	16.5%	1.130	13.0%
Firm Entry Rate	0.136	0.130	-4.40%	0.151	11.0%
Employment Share of Large Firms	0.441	0.450	2.04%	0.403	-8.6%
Top 10% Concentration (Sales)	0.724	0.736	1.60%	0.700	-3.20%
Cash Transfer	-	0.039	-	0.066	-

Table 8: Steady State Comparison of a Tax Credit to Firms' Intangible Investment

entry rate, concentration in terms of employment and sales, and the cash transfer that is needed to finance the tax credit at the stationary equilibrium of both economies. Intangible investment increases due to the policy. An interesting result is that the R&D tax credit even strengthens the results on concentration and prevents firm creation, compared to the benchmark economy. Why is this the case? As Figure 7 shows in Appendix B, firms with higher productivity in producing intangible investment goods are more financially constrained because they rely more on intangible capital for production, and intangible capital is not collateralizable. This means an R&D tax credit has a heterogeneous impact on firms with different intangible capital intensities. Firms with high productivity in producing intangible investment goods, conditioned on other characteristics, are more intangible-capital intensive and will become much less constrained with the implementation of the R&D tax credit, thus benefitting more from the policy. Since they are also large, an R&D tax credit shifts firm distribution toward large firms. Consequently, both the employment share and the market share of large firms increase.

When only SMEs are qualified for R&D tax credits as shown in column 4 of Table 8, the firm entry rate increases and the concentration in terms of the employment share and the market share of large firms declines compared to the benchmark economy. Although R&D tax credit policy targeting only SMEs can induce more firms to enter, viewed through the lens of my framework, this may not

be an optimal policy in terms of consumption utility. It is the rise of intangible capital that promotes economies of scale, due to its non-rivalry property, and increases output. Since the largest proportion of intangible capital is in the hands of large firms, a policy that favors relatively small firms would have a negative impact on output and welfare. When comparing the two policies, we can see that although they contribute to the increase in the intangible investment as a share of value-added to the same magnitude as the baseline, the policy in Case I only needs around half of the cash transfer of the Case II policy to be financed.

The result further indicates that the rise in concentration may not be a negative outcome: policies aimed at stimulating R&D investment without particularly targeting SMEs indeed increase concentration but simultaneously improve the welfare (consumption). Since large and highly productive firms promote economies of scale and contribute to a disproportionately large fraction of the total final output, policies that favor those firms can improve the welfare in terms of consumption in a highly competitive market. This result is consistent with Bighelli et al. (2020), who use a firm-level panel of 19 European countries and find that the increasing concentration within sectors is associated with rising sector-level productivity and that more efficient firms gain increasing market shares.[41]

6   Conclusion

In this paper, I propose a general equilibrium framework of firm dynamics, highlighting the role of intangibles, that can potentially account for two important macroeconomic trends in the U.S. business sector over the past three decades: (i) declined measured labor income share; and (ii) increased concentration in large firms in terms of employment and sales at the national level. I show that a significant part of these phenomena can be explained by a secular and drastic fall in the relative price of intangible investment goods, particularly software, manifested as an increase in the aggregate productivity in producing intangible investment goods relative to producing consumption/physical investment goods, which I call an "intangible-investment-specific technical change" (IISTC).

In my theory, individual firms operate a nonstandard Cobb-Douglas technology to produce both types of goods with heterogeneous productivity, and they accumulate intangible capital by producing intangible investment goods in-house. Due to the non-rivalry property of intangible capital, highly

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[41]However, my results do not necessarily mean, in reality, government should promote policies that favor large firms, considering that my model does not feature market power. As indicated in DeLoecker, Eeckhout, and Mongey (2021), when taking both the market structure and technological change into consideration, the impact of such policies on welfare is ambiguous. On the one hand, that some firms become more productive and more resources reallocate to those firms leads to an increase in welfare. On the other hand, welfare may decrease if deadweight losses are huge resulting from change in market structure.

productive firms operating at a sufficiently large scale demonstrate increasing-returns-to-scale technology and are able to produce more efficiently as they accumulate more intangible capital. These firms benefit disproportionately from the IISTC. Cross-sectionally, my model can replicate the negative correlation between firm-level labor share and firm size well from Autor, Dorn, Katz, Patterson, and Van-Reenen (2020). Consequently, the IISTC shifts firm distribution toward large firms with low shares, which leads to simultaneously rising concentration and declining labor share.

By reconciling the empirical facts regarding labor share and concentration using both aggregate and census data, my work emphasizes the importance of a comprehensive approach that links changes in micro-level heterogeneity to macro-level outcomes when analyzing the drivers of prominent empirical trends in the U.S. business sector.

Taken together, my results have several broad implications. First, they highlight the role of intangible investment in accounting for the trends in measured labor share. The decline in the measured labor share can reflect both technological change and improved measurement. Because intangible investment is largely unobservable, its measurement is challenging (Bhandari and McGrattan (2019); Corrado et al. (2016); Corrado, Hulten, and Sichel (2005); Karabarbounis and Neiman (2018); McGrattan and Prescott (2010b)). However, since an essential aspect of the U.S. macroeconomic model is the factor distribution of income which relies explicitly on the measurement of intangible investment and has important implications on other macro trends such as measured TFP,[42] future research efforts should be devoted to reasonably defining the boundaries of intangibles, accurately measuring intangibles at both firm level and aggregate level and determining the factor distribution of intangible capital rents.

Second, since the focus of this paper is to study the driving forces in the evolution of the labor share of the U.S. business sector rather than its consequences, my model concentrates on the firms' side and abstracts from household heterogeneity. However, over the past five decades, U.S. households have experienced rising inequality and uneven growth (Heathcote, Perri, and Violante (2010, 2020); Lippi and Perri (2019); Moll, Rachel, and Restrepo (2019)). Understanding the potential implications of the secular change in the factor distribution of national income on rising inequality as well as the patterns of uneven growth can be another avenue for future research.

Third, my results also indicate that a large fraction of the slowdown of business dynamism (e.g.,

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[42]For example, in Guvenen et al. (2018), they show that increasingly common profit-shifting practices related to intangible capital, in which a multinational enterprise effectively underprices intangible capital when "sold" from one of its entities in a high-tax jurisdiction to another of its entities in a low-tax jurisdiction, have non-trivial impacts on national statistics especially the aggregate productivity growth rates.

the decline in firm entry) and rising concentration are the natural consequences of technological advances such as falling software prices that favor large, intangible-intensive, and highly productive firms. These firms are more adaptable to a transition toward a more intangible-intensive economy, thereby increasing their efficiency and advancing their market share. In other words, when the decline in the relative price of intangibles is matched to the data, my model is able to account for a significant part of the increase in concentration without involving markups. Viewed through the lens of my model, policies such as an R&D tax credit that favor large firms can improve welfare, since large firms own the largest proportion of intangible capital in the economy, and it is the rise of intangible capital that promotes economies of scale, due to its non-rivalry property, and increases output and welfare. However, this does not necessarily mean that government should promote policies that favor large firms. A more comprehensive assessment of welfare should take both technological change and market structure into consideration, as in DeLoecker, Eeckhout, and Mongey (2021).

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Appendix A: Data and Measurement

In this appendix, I describe the data used in constructing the empirical moments to discipline the model and the intangible-investment specific technical change (IISTC). Subsection A.1 provides the details of the sources and construction of the aggregate data series. In particular, I discuss how I construct the relative price of intangible investment goods in terms of consumption/physical investment goods in Subsection A.1.2. I also show that my model can reproduce the national accounts table in Subsection A.1.3. Subsection A.2 is about the firm-level data I use. In particular, I discuss how I measure intangible capital at the firm-level to be consistent with BEA in subsection A.2.1.

A.1 The Construction of Aggregate Data Series

All the aggregate series are retrieved for the period 1975-2016.[43] There are three sources of data that I use:

National Income and Product Accounts (NIPA-BEA)   NIPA 1.7.5, NIPA 1.12, NIPA 1.13, NIPA 1.14, NIPA 2.3.3, NIPA 2.3.5, NIPA 5.3.4, NIPA 5.3.5

Fixed Assets Accounts (FAT-BEA)   FAT 1.1, FAT 1.3, FAT 2.1, FAT 2.4, FAT 4.7

Flow of Funds   Table L.102: Aggregate balance sheet data for the U.S

A.1.1 Depreciate Rate by Type of Capital

I construct the net stock of capital and depreciation of capital for traditional physical capital and for intangible capital (corresponding to IPP capital in BEA). Since I focus on the private sector only, the net stock of traditional physical capital is the private sector nonresidential structures, equipment, and residential capital. The net stock of IPP capital is only in nonresidential.

The net stock of capital by type of capital (BEA-FAT 1.1, 2.1):

Private IPP: K^{IPP}

Private physical: $K^T = K^{P,ST,NRes} + K^{P,EQ,NRes} + K^{P,Res}$

The depreciation by type of capital (BEA-FAT 1.3, 2.4):

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[43]Choosing 1975 as the starting year is to ensure better estimates and consistency for intangible investment (because 1975 is the first year that the Federal Accounting Standards Board (FASB) requires firms to report R&D), and hence, the measured factor shares of income

Private IPP: DEP^{IPP}

Private physical: $DEP^T = DEP^{P,ST,NRes} + DEP^{P,EQ,NRes} + DEP^{P,Res}$

The capital depreciation rate by type of capital is then:

$$\delta_I = \frac{DEP^{IPP}}{K^{IPP}}$$

and

$$\delta_T = \frac{DEP^T}{K^T}$$

A.1.2 Relative Price of Intangible Investment

I construct the relative price of investment in traditional physical capital and in intangible capital (corresponding to IPP capital in BEA). The price of the bundle of consumption/physical investment good is the numeraire.

I first construct the price index for consumption P_t^C. Let P_t^C be the price index for nondurable goods (ND) and service good (SD) i in year t, computed as the ratio between nominal consumption of good i, C_{it}, and the quantity index of good i, $Q_{C_{it}}$, i.e. $P_t^C = \frac{C_{it}}{Q_{C_{it}}}$, for $i \in \{ND, SV\}$. Let $s_t^{C_i} = \frac{C_{it}}{C_{NDt} + C_{SVt}}$ be the corresponding nominal share of good i in period t. All the variables are from NIPA 2.3.3 and 2.3.5. Denote the growth rate of a variable x_t to be $\lambda(x_t) = \frac{x_t}{x_{t-1}} - 1 \approx ln\left(\frac{x_t}{x_{t-1}}\right)$. Then, the growth rate of the *Törnqvist* price index for consumption is

$$\lambda\left(P_t^C\right) = \sum_i \frac{s_t^{C_i} + s_{t-1}^{C_i}}{2} \lambda\left(P_t^{C_i}\right)$$

The level of the consumption price index is recovered recursively:

$$P_t^C = P_{t-1}^C \left[1 + \lambda\left(P_t^C\right)\right]$$

where P_0^C is normalized to 1 at the initial period.

Second, I construct the price of investment in traditional physical capital including structures and equipment. For price of investment in structures P_t^{ST}, I use price index for consumption P_t^C constructed in step 1 as a proxy. In computing the price index of equipment investment, I use a *Törnqvist* price index for private residential equipment investment, $P_t^{EQ,Res}$, and private non-residential equipment investment, $P_t^{EQ,NRes}$, from NIPA Table 5.3.4. Let $s_t^{EQ,Res}$ and $s_t^{EQ,NRes}$ be the share of private residential and non-residential equipment investment of total equipment

investment using data from NIPA Table 5.3.5. Then the growth rate of the price index of equipment is

$$\lambda\left(P_t^{EQ}\right) = \left(\frac{s_t^{EQ,Res} + s_{t-1}^{EQ,Res}}{2}\right)\lambda\left(P_t^{EQ,Res}\right) + \left(\frac{s_t^{EQ,NRes} + s_{t-1}^{EQ,NRes}}{2}\right)\lambda\left(P_t^{EQ,NRes}\right)$$

Then in computing the price index of traditional physical investment, I use a *Törnqvist* price index again for structures and equipment constructed above. The growth rate of the price index of the traditional physical investment is given by:

$$\lambda\left(P_t^T\right) = \left(\frac{s_t^{EQ} + s_{t-1}^{EQ}}{2}\right)\lambda\left(P_t^{EQ}\right) + \left(\frac{s_t^{ST} + s_{t-1}^{ST}}{2}\right)\lambda\left(P_t^{ST}\right)$$

where s_t^{EQ}, s_t^{ST} are the share of equipment and structures investment of total traditional physical investment using data from NIPA Table 5.3.5.

Third, I construct the price of consumption/physical investment bundle using results in first two steps. Let $s_t^C = \frac{C_{NDt}+C_{SVt}}{C_{NDt}+C_{SVt}+invest_{EQ}+invest_{ST}}$ be the corresponding nominal share of consumption goods in the sum of consumption and traditional physical investment in period t where $invest_{EQ}, invest_{ST}$ are nominal investment in equipment capital and structures capital respectively from NIPA Table 5.3.5. Similarly, Let $s_t^T = \frac{invest_{EQ}+invest_{ST}}{C_{NDt}+C_{SVt}+invest_{EQ}+invest_{ST}}$ be the corresponding nominal share of traditional physical investment in the sum of consumption and traditional physical investment in period t. Using a *Törnqvist* price index again for consumption and traditional physical investment constructed above. The growth rate of the price index of the consumption/physical investment bundle is given by:

$$\lambda\left(P_t^{C,T}\right) = \left(\frac{s_t^C + s_{t-1}^C}{2}\right)\lambda\left(P_t^C\right) + \left(\frac{s_t^T + s_{t-1}^T}{2}\right)\lambda\left(P_t^T\right)$$

Then the level of the price indices of consumption/physical investment bundle is recovered recursively as

$$P_t^{C,T} = P_{t-1}^{C,T}\left[1 + \lambda\left(P_t^{C,T}\right)\right]$$

Fourth, I construct the price of investment in IPP, which is only available for non-residential investment. I use the price index for IPP Investment P_t^I available in NIPA Table 5.3.4. As in constructing P_t^C, normalize the price index of the IPP investment to 1 at the initial period as well.

Finally, the relative price of investment (using the consumption/physical investment as nu-

meraire) is defined as

$$p_t = \frac{P_t^I}{P_t^{C,T}}$$

A.1.3 National Accounts

The national accounts for the model can be expressed mathematically in terms of shares of income and product, where total income is $Y_a = Y_b + \bar{Y}_{nb}$ where $Y_b = Y + PX_I = \int y d\varphi^p + \int px_I d\varphi^p$ represents business income and $\bar{Y}_{nb}$ denotes nonbusiness income. $\bar{Y}_{nb}$ and government expenditure $\bar{G}$ are included as exogenous source of income.

Gross Value-added (GVA) of corporate sector is defined by:

$$GVA = \underbrace{(1-\tau)Q_C}_{After\,tax\,Gross\,Output} - \underbrace{P_{Mt}M_t}_{Intermediates} + \underbrace{\tau Q_C}_{Net\,taxes\,on\,production} = \underbrace{\int_{y+px_I\geq\bar{v}} y d\varphi^p + \int_{y+px_I\geq\bar{v}} px_I d\varphi^p}_{Final\,output} + \tau Q_C$$

Labor income of corporate sector is defined by:

$$WL_C = w \int (l_1 + l_2 + \kappa_o)\, d\varphi^p + \tilde{w} \int_S \kappa_e d\varphi^e$$

Net Operating Surplus (NOS) of corporate sector:

$$NOS = \int_{y+px_I\geq\bar{v}} [y + px_I - w(l_1 + l_2 + \kappa_o) - (r + \delta_T)k_T]\, d\varphi^p$$

Identity:

$$\underbrace{Y_C + PX_{IC} + \tau Q_C}_{GVA} = \underbrace{WL}_{labor\,income} + NOS + \underbrace{\overbrace{\int \delta_T k_T d\varphi^p}^{depreciation} + \overbrace{\tau Q_C}^{net\,prod.\,taxes}}_{Capital\,income}$$

Then corporate business income/total business income is given by:

$$\frac{Y_C + PX_{IC} + \tau Q_C}{\underbrace{(Y_b + \bar{Y}_{nb})}_{Y_a}}$$

The national accounts of this economy can be summarized in Table 9.

		Data	Model
Income Shares			
Business income	$\left(\int y d\varphi^p + \int px_I d\varphi^p + \tau Q_C\right)/Y_a$	0.791	0.791
Corporate business income	$\left(\int_{y+px_I \geq \bar{v}} y d\varphi^p + \int_{y+px_I \geq \bar{v}} px_I d\varphi^p + \tau Q_C\right)/Y_a$	0.608	0.608
Corporate labor income	wL_C/Y_a	0.385	0.385
Net operating surplus	NOS/Y_a	0.100	0.124
Consumption of fixed capital	$\left(\int_{y+px_I \geq \bar{v}} \delta_T k_T d\varphi^p\right)/Y_a$	0.075	0.051
Production taxes	$\tau Q_C/Y_a$	0.048	0.048
Noncorporate business income	$\left(\int_{y+px_I < \bar{v}} y d\varphi^p + \int_{y+px_I < \bar{v}} px_I d\varphi^p + \tau Q_{NC}\right)/Y_a$	0.183	0.183
Nonbusiness income	$\bar{Y}_{nb}/Y_a$	0.209	0.209
Product Shares			
Private Consumption	C/Y_a	0.612	0.632
Private Investment	$\left(K_T - (1-\delta_T)K_T + PX_I\right)/Y_a$	0.182	0.144
Gov't Investment + consumption	$\bar{G}/Y_a$	0.206	0.219

Table 9: National Account Shares, Data and Benchmark Model

A.2 Firm-level Data

For measurement of intangible capital at the firm-level, I use Compustat North America-Capital IQ, which provides annual accounting data for publicly listed U.S. firms. This data set fits our purpose well because firm-level R&D investment data are available and because it is well-suited to study U.S. firms' financial aspects due to its rich firm characteristics and industry information. I exclude foreign firms, government-sponsored firms, public utilities and financial firms, as is commonly done in the investment literature. I also exclude mergers, acquisitions, and observations with extreme values. To be consistent with the aggregate data, I focus on the period 1975 - 2016. After following the standard data cleaning procedures (see, for example, Imrohoroglu and Tuzel (2014); Ottonello and Winberry (2018)), I end up with 14,734 firms in total and 153,505 firm-year observations. The representativeness of the dataset is fairly good: total assets Compustat/Flow of Funds (non-financial corporate sector) ranges from 50 to 75%. Again, since I focus on all the employer firms in my model,I filter a subset of firms in my model based on firm size with the criterion specified in the main text to target moments constructed using Compustat data.

A.2.1 Measurement of Intangible Capital at the Firm-level

Existing literature attempts to measure intangible capital either directly or indirectly. The indirect approach is to construct a proxy using aggregate stock market or national accounting data (e.g. Karabarbounis and Neiman (2018); McGrattan and Prescott (2010b)). These approaches measure intangibles as unexplained (by physical capital) residuals of stock market value or firm productivity.

The other approach is to construct aggregate measures of the different components of intangible capital directly (Corrado, Hulten, and Sichel (2005)) using a wide range of aggregate datasets including NIPA, the Services Annual Survey (SAS), and the surveys of employer-provided training from BLS. The aggregate data on intangibles considered in this paper is measured following the method developed by Corrado, Hulten, and Sichel (2005), henceforth, CHS. The biggest advantage of this method is that it includes the most types of intangible assets including the very firm-specific human and structural resources, in addition to software, R&D, and artistic originals which have been included into the national accounts. In general, CHS's method includes three categories of business intangibles: (1) computerized information, which is a firm's knowledge embedded in the computer programs and computerized databases; (2) innovative property, which is a firm's knowledge acquired through scientific R&D and non-scientific inventive and creative activities; (3) economic competencies, which is a firm's knowledge embedded in firm-specific human and structural resources, including brand equity and on-the-job training.

For the purpose of accounting for the BEA-measured income shares, I target the BEA-measured intangibles (or IPP called by the BEA) on the aggregate. To be consistent with the BEA method on the firm-level, I construct a measure of intangible capital including software and R&D using Compustat data. The difficulty is that the capital that is created by investments in intangible assets such as R&D are only expensed, thus not being reported on firms' balance sheets. Following the method developed by Peters and Taylor (2016) and Falato et al. (2018), the essential idea to overcome this difficulty is to capitalize expenses related to intangible assets consistent with BEA.

More specifically, the replacement cost of knowledge capital is measured by capitalizing R&D expenditures using perpetual inventory method with depreciation rate of 20%.

Since firm-level expenses on software are not available, I construct an industry-level measure to approximate the intangible assets on them. The BEA classification features 63 industries. I match the BEA data to Compustat firm-level data using SIC codes, assuming that, for a given year, firms in the same industry have the same shares of intangible assets on software. I construct measures of software shares for industry l in year t as

$$software_{l,t} = \frac{IPP_{l,t}^{BEA} \times \frac{software_t^{BEA}}{IPP_t^{BEA}}}{FixedAsset_{l,t}^{BEA}} FixedAsset_{l,t}^{Compustat}$$

where $FixedAsset_{l,t}^{Compustat}$ are total assets in industry l in year t. Since data on the fixed assets of software are only available at the aggregate level rather than the industry level but intellectual

coefficient estimate	Data	Model
α	-0.078***	-0.043***

***$p < 0.01$

Model simulated starting with entrants distribution for 50,000 firms and 100 years

Table 10: Financing patterns

property products are, I use the economy-wide ratio of software to IPP multiplied by the IPP at the industry level to approximate the industry-level intangible assets on software. The BEA data comes from Fixed Assets Accounts Tables (FAT) Table 3.7I. Informational capital is constructed by capitalizing expenditures on software with a depreciation rate of 31% following BEA.

Knowledge capital (in terms of the replacement cost), information capital, and the on-the-balance-sheet intangibles consists of the intangible capital stock on the firm-level. When a firm purchases an intangible asset externally, for example, by acquiring another firm, the firm typically capitalizes the asset on the balance sheet as part of Intangible Assets, which equals the sum of Goodwill and Other Intangible Assets. The asset is booked in Other Intangible Assets if the acquired asset is separately identifiable, such as a patent, software, or client list[44]. Acquired assets that are not separately identifiable, such as human capital, are in Goodwill. When an intangible asset becomes impaired, firms are required to write down its book value. There is debate about whether on-the-balance sheet "intangibles" should be added into the measure of intangible capital on the firm-level or not. Following Peters and Taylor (2016), I keep Goodwill in Intangible Assets in my main analysis, because Goodwill does include the fair cost of acquiring intangible assets that are not separately identifiable. However, it should be noted that due to the inclusion of goodwill, the item picks up over-payment in mergers & acquisitions (M&As). Hence, I also try excluding Goodwill from external intangibles as a robust check.

In general, my resulting estimate for the ratio of intangible to tangible capital over the past decades is comparable to the estimate based on BEA.

Next, I discuss two empirical patterns I generate from Compustat and compare the corresponding patterns generated from my model with them, which serve as validations.

First, I find that intangible-intensive firms tend to rely less on external debt finance. More

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[44]But this variable "intangibles-Other" variable in Compustat that is net of goodwill is only available in Compustat since 2000

	Dependent variable: market share			
	(A)	(B)	(C)	Model
Compustat intangible share	0.105***	0.025***	0.014***	0.258***
Industry×Year F.E.	Yes	No	No	-
Firm F.E.	No	Yes	Yes	-
Year F.E.	No	No	Yes	-

***$p < 0.01$

Model simulated starting with entrants distribution for 50,000 firms and 100 years

Table 11: Intangibles and market share

specifically, I run the following OLS regression:

$$\frac{Net\ Debt_{it}}{Asset_{it}} = \alpha \frac{Intangible\ Capital_{it}}{Asset_{it}} + \beta X_{it} + Year_{dummy} + Industry_{dummy} + \varepsilon_{it}$$

where X: a set of control variables including Tobin's Q, physical investment/assets ratio, firm size, dummy variables for positive dividend payout. I report the results in Table 10 with comparison between model and data.

Second, I find that a firm's market share in its industry is higher when its intangible capital to total assets is higher, and this relationship holds between firms of the same industry, within firms over time, and controlling for year effects. This is in line with Crouzet and Eberly (2019) although they measure intangible capital at the firm-level different from mine [45]. I report results in Table 11.

A.2.2 Other Firm-level Moments

For firm distribution, I focus on all the employer firms in U.S. business sector. For statistics related to firm entry, exit, job creation and destruction, firm size and age distribution, I calculate directly from the Business Dynamic Statistics (BDS), which are compiled from the Longitudinal Business Database (LBD). The LBD is a confidential longitudinal database of business establishments and firms starting from 1976. For any moments I need to rely on LBD, I draw them from the existing literature with corresponding years.

As I have mentioned in the main text, I admit that there exists discrepancy between the coverage of BEA-business sector, which covers both employer and non-employer firms and the coverage of BDS/LBD, which only covers employer firms. However, as shown in Table 12, since non-employer firms do not contribute to the total employment and only takes a very small portion of total sales

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[45]Crouzet and Eberly (2019) don't capitalize R&D expenses and software expenses. That is, they only consider intangibles on the balance sheet.

	Data Source	Number of Employees
	BDS 2007	119,627,020
	SBO PUMS 2007	30,465,820
	Compustat 2007	73,434,000
	BEA 2007 (all private domestic industries)	118,944,000

Data Source	Employer Firms (number)	Employer Firms' receipts ($1000)	Nonemployer Firms (number)	Nonemployer Firms' receipts ($1000)
SBO 2007	5,287,344	10,015,051,752	21,104,893	932,487,829
BDS 2007	5,240,019	-	0	0

Table 12: Coverage of Firms in Multiple Datasets

Note: The SBO covers all nonfarm businesses filling IRS tax forms as individual proprietorships, partnerships, or any type of corporation with receipts of $1,000 or more. However, businesses classified in the SBO as publicly owned are not included in the PUMS version.

(which is 2.48% based on SBO 2007), I assume BEA-business sector and BDS/LBD cover the same firms.

Change in Category	Data (pp.)	Model (pp.)
Labor share (post-2013-revision)	-4.6	-2.3
Labor share (pre-1999-revision)	-2.4	-1.0
Saving flows/GVA	+6.0	+4.8
Annual Firm Entry rate	-4.5	-1.5
Employment Share of Large Firms (500+)	+4.7	+4.4
Employment Share of Mature Firms (11+)	+13.7	+6.0
Top 10% concentration (Sales)	+5.3	+4.5
Average firm size	+12.2	+9.0
Labor productivity dispersion	+8.0	+4.4
Labor productivity gap between frontier and lagged firms	+20.00	+6.10

Table 13: Aggregate Implications: Additional Results

Appendix B: Empirical Facts

In this section, I summarize the empirical facts, mainly declined labor share and increased concentration, that my paper attempts to explain, together with some additional facts regarding business dynamism that my model can account for as well.

To start with, I briefly discuss the two measures of labor share I consider in the paper. I focus on the labor share in corporate sector for three reasons. First, corporate sector has a much cleaner measurement of labor income and profit compared to non-corporate sector. For example, for non-corporate sector, part of proprietor's income should be contributed to labor income since entrepreneurs contribute their own hours to the business. Second, the corporate share of U.S. GDP is fairly stable. Third, very large firms are usually corporates. Based on census data, 20% of firms in the U.S. are corporations, which account for 90% of total sales in early 1980s (Dyrda and Pugsley (2019)). Since this paper is about jointly explaining the trends for labor share and industrial concentration, focusing on the corporate sector would be appropriate.

The key difference between the two measures of labor share considered in this paper is whether intangibles are capitalized or not.[46] BEA has done two comprehensive revisions of the national income and product account (NIPA) on the capitalization of intellectual property products (IPP). In 1999, the 11th BEA revision capitalized software expenditures. In 2013, the 14th revision started treat R&D expenditures and artistic originals as investment in the form of durable capital. In Figure 6, I plot the evolution of the two measures of labor share respectively. I choose the starting year to

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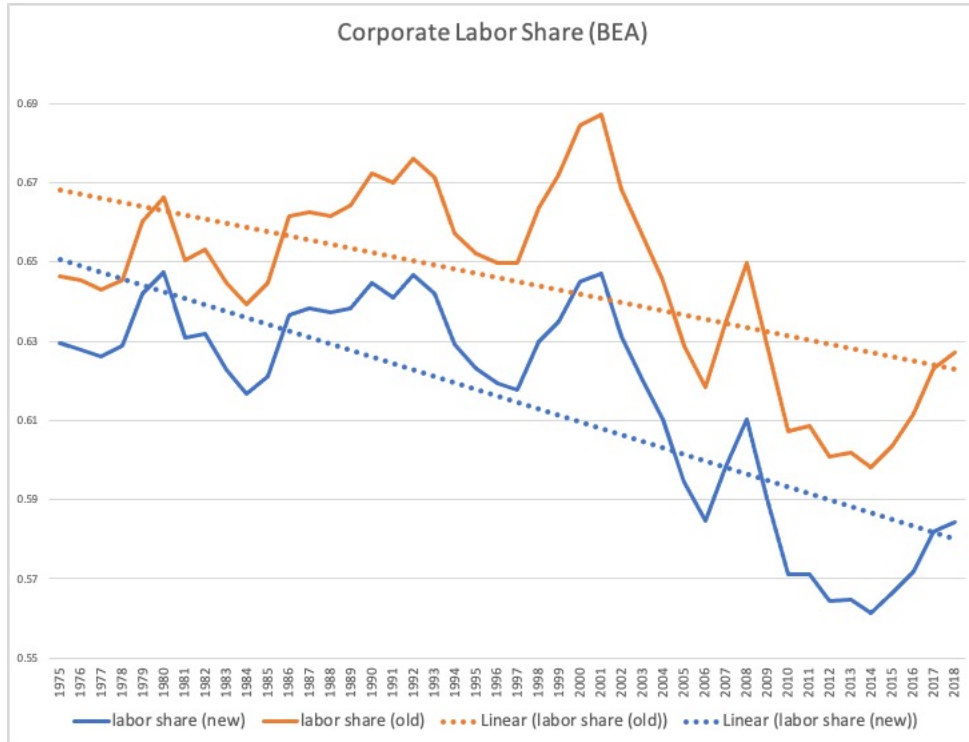[46]For a more detailed discussion, see Koh, Santaeulalia-Llopis, and Zheng (2020).

Figure 6: Measurement of Labor Share

be 1975 because it is the first year that the Federal Accounting Standards Board (FASB) required firms to report R&D. Based on the data, the magnitude of the decline in labor share of the post-2013 revision of BEA is larger than that of the pre-1999 revision. This is consistent with the increased intangible investment as a share of gross value-added, which implies that the drop in the aggregate labor income share would be smaller if intangibles are not treated as final output.

To summarize, I focus on the following regularities, most of which occur during the the past three decades:

1. The labor share of gross value-added has gone down.

2. The employment share of large firms (500+ employees) has risen.

3. The employment share of mature firms (11+ years) has risen.

4. Market concentration in terms of top 10% has risen.

5. Firm entry rate has declined.

6. Productivity dispersion of firms has risen. Similarly, the labor productivity gap between frontier and laggard firms has widened.
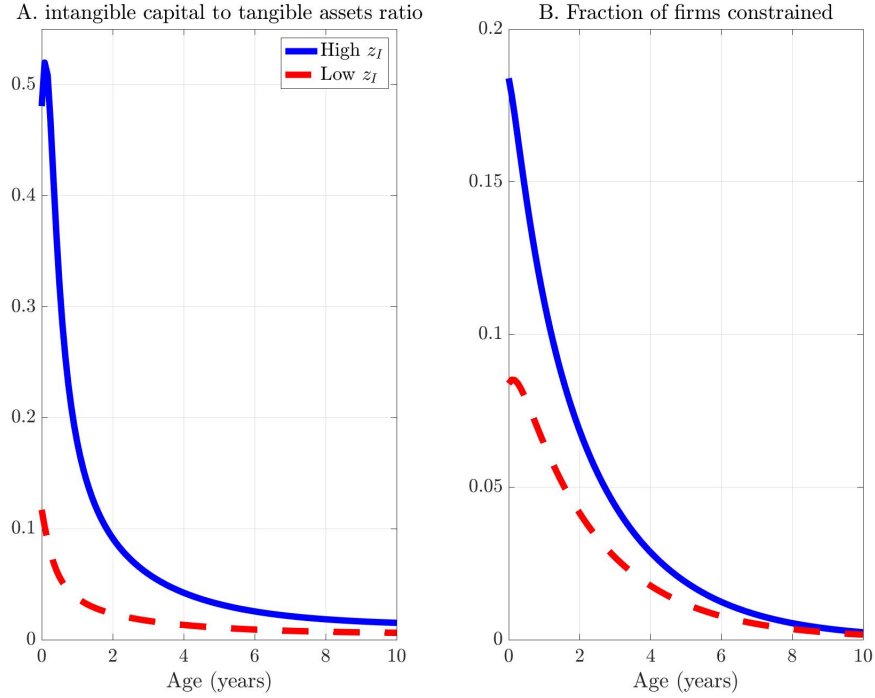
Figure 7: Life Cycle Dynamics of Firms: Additional Results

7. The corporate saving flows relative to gross value-added has risen, together with weaker physical investment.

The rise of corporate saving is due to the non-collateralizability of intangible capital. Since the IISTC leads to greater importance of intangible capital in firms' production, firms accumulate more intangible capital. On the other hand, since intangible capital cannot be used as collateral for firms to borrow, firms need to rely more on internal financing rather than external financing. This is the channel where the IISTC results in increased firms' saving flows relative to the total income on the aggregate level. Additional results on the cross-sectional life cycle dynamics of firms can also shed light on this point. In Figure 7, we can see that firms with high productivity in producing intangible investment goods are more intangible-capital intensive and are also more financial constrained. As firms become mature, they can gradually get rid of the collateral constraints.

I summarize the results on how my model accounts for the additional empirical facts driven by the IISTC besides labor share and concentration in Table 13.

Appendix C: Computational Algorithm

I describe the solution method for my long-run GE economy in this section. The usual nested fixed point approach is extended in order to accommodate the additional features of my model. That is, the essence of our approach is to guess a set of prices, compute decision rules (given prices) to simulate the economy, and finally verify whether those are the equilibrium prices.

Specifically, I execute the following steps:

1. Make an initial guess for the wage $\tilde{w}$. With specifying the upper bound and lower bound for wage, w_0^u, w_0^l, the initial guess can be $\frac{w_0^u + w_0^l}{2}$. Due to the risk neutrality of households, the real interest rate is $r = \frac{1}{\beta} - 1$ at the steady state.

2. Solve both incumbent firms' and entrants' dynamic programming problem described in Section 3 at the prices $\tilde{w}, r$. The general procedure involves using a nested vectorized golden search method to solve the optimization problems and find the policy functions $\left(k_I'(s), a'(s)\right)$. More specifically, I solve the optimization problem in two steps: (1) solve the policy function for $a'(s)$ given $k_I'(s)$. I first solve (k_{T1}, l_1) which is the solution to a static problem of firms. I then define a cash-in-hand variable $cih = y - (r + \delta_T)k_{T1} - \tilde{w}l_1 + (1 + r)a - \tilde{w}\kappa_o$, and solve k_{T2}, l_2 according to the production technology for intangible investment goods (equation 2) given k_I', z_I, and prices R_T and $\tilde{w}$. Note that the effective rental rate of physical capital can be different from $(r + \delta_T)$ since some firms' collateral constraint are more binding than the other, so I also need to solve the value of the Lagrange multiplier of the collateral constraint, call it ζ faced by each individual firm using a bisection method. Next, check two things: whether $cih - wl_2 - R_T k_{T2} \geq 0$ or not and whether $\zeta > 0$ or not. Depending on the answers to the two checking questions, we are going to discuss three cases. Case I: $cih - wl_2 < 0$. In this case, we simply set $a' = 0$ and $d = cih - wl_2 - R_T k_{T2}$. Negative dividend payment serves as a penalty. Case II: $cih - wl_2 - R_T k_{T2} \geq 0$ and $\zeta > 0$. In this case, firms are constrained, thus paying zero dividend, i.e. $d = 0$. Then $a' = min(cih - wl_2 - R_T k_{T2}, amax)$. Case III: $cih - wl_2 - R_T k_{T2} \geq 0$ and $\zeta = 0$. In this case, firms are NOT constrained and thus need to pay the dividend. Use golden search to solve for $\left(a', d\right)$. (2) after expressing a' in terms of k_I', solve for k_I' using golden search again.

3. Using the policy functions for both incumbents and entrants to find the stationary distribution using the method developed by Young (2010).

4. Given the stationary distribution and policy functions of firms, compute the aggregate labor demand LD.

5. Compare the aggregate labor demand LD at prices $\tilde{w}, r$ with the inelastic labor supply $\bar{N}$. If $|LD - \bar{N}| < tolerance$, then we are done. Otherwise, we employ a **bisection price updating scheme**. More specifically, we update the guess for wage based on the following rule: if $LD > \bar{N}$, set the lower bound equal to the lower bound the same as before, $w^l = w^l{}_{old}$ and the upper bound equal to the wage in the current iteration, $w^u = \tilde{w}$; if $LD < \bar{N}$, set $w^l = \tilde{w}$ and $w^u = w^u{}_{old}$. Then a new guess for wage is $\frac{w^u + w^l}{2}$ and return to step 2.